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ATTRIBUTION OF CLIMATE CHANGE TO THE EXTREME SNOW FALLS OF MAZANDARAN PROVINCE

(stochastic climate modeling using Lorenz-63)

A dissertation

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CHAPTER I

"Concepts without intuition are empty,

intuition without concepts is blind".

Immanuel Kant (1724–1804)

Introduction

1.1 Importance of this study

One of the main reasons why we are interested in studying rare events is because they are usually associated with unwanted incidents of high-potential environmental, social, financial, infrastructural, or technological impacts [24]. The regular weather patterns come to be irregular and intense. Then devastation comes along. Is that a common event in the climate system? is that a human induced one?

Extreme snow in a region such as Mazandaran province of Iran is not a regular event in winter time. Although having snow fall on that region is a very rare event. However, in the south mountain areas of the province snow fall is a common winter time precipitation. There exists no universal definition for the extreme snow fall but it is defined by the AMS in amount of ~5 inch. That is this amount is severely affected by the geographical and geomorphological properties of the region of interest. For the region of our case study the definition of the AMS is defined since the region is not prone to snow fall, let alone heavy snow! Therefore, this definition is seeming to be logical.

Extreme events study is an active area with several different approaches specially in the context of detection and attribution research. As other branch of research field it also uses primarily definitions. Propositions, theorems and concepts. Since the language of nature is mathematics then the language of the climate change and its detection and attribution of extreme events is also mathematics. But the climate system and its fluctuations is a physical system at first place then in describing it issues we need the language of mathematics. Namely there is need to bring the situation down and define the system of climate in general in a correct physical manner then define the mathematical terms Also in a correct manner and after that solve the proposed problem with a suitable solution. This is a common way in solving physical problems and climate system basically is a physical system. In atmospheric sciences the is no laboratory to do your experiments but your model. Then establishing a climate and weather model is in a primary importance in the context of extreme event attribution field.

1.2 The questions

This need caused to bring the situation down about the models of climate in a general way with respect to the physical process in the climate system. In a general way as the climate system is a highly complex physical system since it is an open system not a closed system and it is in active interaction among its sub- systems and its environment as a planet. The Figure of (1.1) are showing this fact very clearly.

Fig.1.1 – The planet earth as blue particle of dust floating in sun beams and cosmic dust. This is the most distance picture from the earth, and of earth by voyager (1). Courtesy NASA.

The Fig 1.1 is a fabulous pic among the whole history of human kind in this planet. The Mother Earth. But the point of this pic for this case of study is the Hamiltonian and Lagrangian system of equations for describing the physics of such system especially about its climate system is not seems to be logical. But as far as this study seek in the literature and other sources the moving from current generation of climate models to the models based on modern physics foundation instead of classical physical foundations is on progress.

As mentioned above the extreme snow fall in Mazandaran province is a rare event in region which is not prone to snow fall the main reason could be the Mediterranean water cycle of the Caspian Sea and also its 1000 deep in the south part (more explanation in chapter 2). But the aim of this study is not the synoptic or dynamic analysis of the extreme snow fall in the region. The purpose of this study is "attribution of climate change to the extreme snow fall of Mazandaran province". This attribution required a sound physical definition of the problem which is done by the literature of reference books and remarkable review paper such as Lucarini et al (2014). A sound philosophical and epistemological definition of the method of study. Along with a sound mathematical definition. These items are described in a compression and critical way in chapter two.

According to the description of chapter two the different aspects of the study become clear, however it takes a long time of several months to do it. Since it is done for the aim of climate system modeling at first it has a general view. This also is the case for the whole work. The reason is that this study used a modern view and a new approach of extreme event attribution. Then the properties of current climate models are not compatible with the situation defined for this case study as particular and the whole climate modeling in general.

1.3 Defining the problem

Climate system is a chaotic, dissipative, non-equilibrium, probabilistic (because of its intrinsic Brownian motion) and then stochastic physicals and dynamical system. But the place of these property in the current climate models is absent. And in addition the case of this study i.e. extreme snow fall of Mazandaran province is also a random rare event by its own. Consequently, the approach of extreme event attribution in this study is fundamental at first place then this foundation is used to establish the method of the study in chapter three. The method can be classified in data assimilation approaches used for climate studies.

Chapter 3 is on the basis of the method at first. The went along with the fundamental mathematics. The foundation of physics of the model. The description of the little 3-D deterministic model which made the counterfactual world and its purebred with white noises as the factual world. The selection of the two parallel worlds instead of a fairy long time such as 100 years and then divide it in two periods, is that as Landau (1979) in his series of theoretical physics in the book of statistical physics declare a concept as "relaxation time" for a system to reach its equilibrium point which is a closed system yet. Since the climate system is an open system so its relaxation time cannot be defined.

1.4 Over view of ongoing chapters,

The causality of Nietzsche (1888) and pearl (2000) combined in chapter 2 and 3 to bring us the concept of "causal attribution of extreme event" to this study. The data which is used in this study is downloaded from NOAA. The data set selected from NCEP-DOE Reanalysis (2) which is an enhanced version of the NCEP Reanalysis (1) model. The error is fixed and updated parametrizing of physical processes. In this respect it is similar the EnkF method which is used in this study to assimilate the PDFs of the prior, posterior and observation to bring us Maximum likelihood 0f the even occurrence (extreme snow fall of Mazandaran province) in terms of the forcing that is defined here as Net Radiation Flux in relation with the mean temperature (air) 2

meter above the ground. Then the level of the selected parameters is in 1000 Hpa since the general view of the study.

In the environment of GIS, the daily mean downloaded data are processed to gain the required data series for the model of Lorenz-63. This model is an experimental model since there is no universal model with the physical condition defined above this study is done on this 3-D dynamical and chaotic model which is describing a convective process. The ODEs of the system of equations of this model is transformed to SDEs system of equations then their corresponding Fokker-Planck equations computed the the required PDFs in evolution of the time with Kernel function which is also common in integral transformations and integration.

Matlab and Mathematica software are used in transition and computation of the data set from GIS, environment. However, Maple is used for some experimental aspect but no directly in the study. Theses is described in chapter in chapter 3 and 4. It is tried to properly connect the whole theoretical and practical process. EnKF is an essential data assimilation method for high dimensional systems. It's effective presentation of the system using ensemble size (k) which is much lower than surrounding state/phase space dimension (d) i.e. K<<d is stayed a mystery yet [143].

The sample spaces Ω and Ω' for the parallel factual and counterfactual worlds are discrete sample space i.e. is finite or countably infinite which is a basis for discretization process in the method of this study. If Ω is a sample space then ω is the events which defined as $\omega \subset \Omega$ and is a set of possible outcomes then if $\omega \in \Omega$ then the event occurred. In probability of causes which rooted in the Bayes' theorem the intersection of the probability in sample space Ω or Ω' is defined as

$$p(\omega_k|\omega) = \frac{p(\omega_k)p(\omega_k|\omega)}{\sum_{k=1}^{n} p(\omega_k)p(\omega_k|\omega)} \tag{1.1}$$

which is the well-known Bayes rule. This allow us to find the probability (likelihood in this study) of the various events which can be the cause of ω to occur. Then this study is on the basis of the "probability of causes". Then the fluctuation in the net radiation flux and mean temperature are the mutually exclusive events that is the cause of the extreme snow fall in Mazandaran province by the analysis of LM-63 by means of EnKF. More details in the chapter 3 and 4.

The whole of this study is an effort to assist develop stochastic climate models and theory. Then this effort is done by the deep fundamental study of the whole related foundations and

then by using a nonlinear state model. The result showed that if the climate system defines correctly on its intrinsic physical basis then we have a chaotic, dissipative, diffusive, probabilistic dynamical system which can be display and interpret as "fractal geometry of nature".

CHAPTER II

Theoretical background

2.1- Core Definitions and literature

According to IPCC (2014) the climate is the highly complex system comprising five principal components: the atmosphere, the hydrosphere, the cryosphere, the lithosphere and the biosphere. interactions between these components makes the climate system to work. The climate is a dynamical system because it evolves in time. This evolution is influenced by its own internal dynamics. Also it is inspired by external forcing. These forcing include volcanic eruptions, solar variations and anthropogenic forcing. The most prominent human induced changes are perturbations in atmospheric composition and land-use alteration [6]. Considering this sort of chaotic system requires definition of uncertainties which "arise not only from initial conditions and forcing scenarios, but also from model formulation". In a current work [8]. "climate scientists themselves recognize the need for work on the theoretical foundations of their discipline" [7]. According to [8] the prediction ability of the whole climate system, is hindered due to deficiency in current governing physical principles. Based on [7] the more efficient way to deal with this problem is defining suitable **notions** for studying different branches of climate science.

The first step is defining your perspective which according to IPCC (2014) are "narrow and broad notions". The former the long term statistical weather conditions and the later the entire components of climate system. Although [9,10,11,12] considered the climate as a dynamical-statistical system. With this notion you can study the climate condition more efficiently as it characterize climate states or regime. This is on the basis of **dissipative a**nd **chaotic** manners of the climate system through the time. Consequently, the first point is detecting a suitable distribution for interested observation then attribute the changes of that distribution to the related dynamical perturbations [13]. Palmer (1999) based on a dynamical systems perspective, says that, the climate change refers to fluctuations in climate system attractor which fallouts from external forcing. Although the most prominent definitions of climate change are statistical/mathematical but, Pielke (2010) in a comprehensive notion view the climate change as "any multi decadal or longer alteration in one or more physical, chemical and/or biological components of the climate system". Therefore, the notion of climate change is closely related to the notions of climate regime i.e. how to think about them [7]. **climate sensitivity** also is

closely related to them. Climate sensitivity delivers standardized techniques for quantifying the climate system response to a change in forcing.

In this notion, equilibrium climate sensitivity is the most widely used notion. an informal definition is the perturbation in the global mean surface temperature that fallout from increment in atmospheric CO_2 concentration. Although this concentration comes from an instantaneous feedback processes towards their equilibrium state within the ocean-atmosphere system (IPCC 2014). In a more formal notion of equilibrium climate sensitivity is defined as;

$$S = \frac{\Delta T}{\Delta R} \tag{1.2}$$

where ΔT is the difference in global mean surface temperature between two statistical **steady conditions** - two conditions with unchanging distributions of mean surface temperature - and ΔR is the difference (transition) in radiative forcing as the cause of transition between two conditions, such as increment in CO_2 or CH_4 concentration. however von der Heydt et al (2016) defined it as the temperature change $S \times \Delta R_{2\times CO2}$ thus it also defined with respect to statistical steady states instead of equilibrium.

According to Wallace and Hobbs (2006) climate feedback is an internal process of the climate system that able to intensifies and diminishes the initial effect of external forcing or internal variability. Actually it is ($\sum f_i$) that indicate climate sensitivity(f_i as feedback factors)[14].That is we say that the feedback factors are additive.

In the case more one feedback factors, on the basis of total derivatives the climate sensitivity also considers other auxiliary variables (y_i) therefore expand the above climate sensitivity equation by the chain rule as

$$s = \frac{dT}{dF} = \frac{\partial T}{\partial F} + \sum_i \frac{\partial T}{\partial Y_i} \frac{\partial Y_i}{\partial F} \tag{2.2}$$

Whereas the term ($\frac{\partial T}{\partial Y_i}$) can be put away in the conditions of absence of feedback(S_0).

These sort of definitions are able to offer an appropriate general mathematical characterization for climate sensitivity. Also as previously noted, they satisfy climate's response to additional radiative forcing. Although these also might depend on the conditions of the climate system [7]. But still the characterization of a particular state is subject to question. Although generally it seems that easily you could found footprints of different sort of dependence in all of different kind of notions.

According to the IPCC (2014), climate variability refers to variations in the mean state and other statistics belong to the conditions of climate system in special and temporal scales. The most knowledge about climate variability come from experiments in numerical models [14]. The basis of our inference is basically the equal boundary conditions for historical data and model run, in addition to considering variations in accordance with seasonal variation and climatological mean in a year by year scales. There exist totally two kind of variability; external an internal [7]. Internal variability is referred to natural variability of the climate system and external to additive forcing(anthropogenic) to this natural process.

However, in methods such as fingerprint approach- as the key approach for attribution of climate change- we assume both of them as additive components of the total climate variability [15]. In fact, it attributes observed variability to the sum of all external and internal forcing. According to such methods, internal variability occurs in the condition of nonexistence of external forcing, then hold them constant [16].

As noted before being external or internal depends on definition of boundary and initial conditions of climate system. The former is 'Values for any variable which affect the system but which are not directly output by the calculations". The latter is a mathematical definition of the climate state at its initiative point projected towards of the period being simulated [20,17,18]. according to Bradley et all (2017) typical boundary conditions for the climate system include, the concentration/amount of greenhouse gases, aerosols and received solar radiation by/in the Earth's atmosphere at a given time. Often they are dabbed as external forcing or conditions. Therefore, they are drivers of climatic change.

Accordingly, the first question is the definition of external forcing and internal forcing that depends on definition of boundary conditions. Another fact is that, external forcing able to perturb natural internal processes, even on reasonably short time scales. For instance, perturbations which is obvious in the magnitude and frequency of oscillations in Atlantic and Pacific oceans - such as El Niño Southern Oscillation (ENSO), the Pacific Decadal Oscillation (PDO), and the North Atlantic Oscillation (NAO), and other so-called modes of variability- [14,17,26]. However, [26] believes that "general chaotic elements in the weather and climate system preclude longer-term statements because these regional patterns are not externally forced". Consequently, the question is that, are the fluctuations in natural internal processes caused the fluctuations in conditions of climate system? This is a new pattern of internal variability whose dependence to external and internal variability is the current question.

Although it might be difficulties for attribution of extreme events to the external forcing in this method, it seems attractive especially in considering external variability such as greenhouse gases or aerosols radiative forcing. although its practicality for risk management is another question while it is based on strong mathematical and theoretical basis.

However, "where processes occur on a smaller scale than the grid, they may be included via **parameterization**, whereby the net effect of the process is separately calculated as a function of the grid variables. For instance, cloud formation is a physical process which cannot be directly simulated because typical clouds are much smaller than the grid. So the net effect of clouds is usually parameterized (as a function of temperature, humidity, etc.) in each grid cell and fed back into the calculation. Sub-grid processes are one of the main sources of uncertainty in climate models"[20].

according to IPCC (2014) Radiative Forcing (W/m^2) quantifies the strength of each drivers as "the energy flux caused by a driver and is calculated at the tropopause or at the top of the atmosphere".

Radiative forcing is defined as

$$F = \frac{\partial Q}{\partial U} \acute{U} \tag{3.2}$$

where U is the concentration

which accompany with the climate feedback parameter defined as

$$a = \frac{\partial Q}{\partial T} \tag{4.2}$$

In words, "the change in energy flux caused by a driver". It commonly is calculated at the tropopause or at the top of the atmosphere [18,19].

For every climate study the first need is the accurate data. These data are from the meteorological observations. Such as surface temperature which exist from 1750 up to know. However, it is not a global coverage, obviously. Therefore, scientists use **proxy data.** These sort of data resulting from observing other natural phenomena such as tree rings, ice cores and ocean sediments. It is the fact that proxy data rises methodological complications. These technical hitches involved statistical processing for these sort of data. because sparsity and high uncertainty are their coral nature. Although they need several inferential steps, far more than climate variable in interest [21].

"Detection of change is defined as the process of demonstrating that climate or a system affected by climate has changed in some defined statistical sense without providing a reason for that change. An identified change is detected in observations if its likelihood of occurrence by chance due to internal variability alone is determined to be small". Attribution is defined as "the process of evaluating the relative contributions of multiple causal factors to a change or event with an assignment of statistical confidence" [IPCC 2013] [22].

Although these definitions raise difficulties because it depends on internal variability. Consequently, "climate change has been detected only if "an observed change in the climate is unlikely to be due to internal variability"[18,20]. Internal variability is due to the internal dynamics of the climate system even when climate change (anthropogenic) is not exist. Therefore, there according to this definition, detection, says internal climate change is not existing. "The ice ages, for instance, would not count as climate change if they occurred because of internal variability". Also it encounters problems in the case relatively short time consideration for climate change as their internal climate change is plausible [21].

Therefore, also in attribution, it excludes factors like internal climate dynamics if detected climate change is not due to internal variability. While they can lead to a change in the climate, which can be an unlucky conclusion [23]. Therefore, it seems that for a suitable definition we must count on the research question. But this definition is widely used in literature by the researchers there in the case of our study we also use it for our basic initiative points.

2.2- Philosophy and Epistemology

This field of this study is challenging with "generating causal information about episodes of extreme weather or unusual climate conditions". This is based on the *Casual Counterfactual* theory for the attribution of weather and climate events. Although roots in traditional cause and effect theory [1]. According to Stott et al (2013), the needs for public broad casting, legal process, adaptation to climate change, and related scientific improvements are the common challenges in attribution of extreme events [2]. However, for dealing with the deficiency of clear semantics for defining or interpreting causal factors this study is depends on ideas of Nietzsche in scientific evaluation methods.

Friedrich Nietzsche in the book three of "the will to power" titled as "the principles of new evaluation", advise valuable insights about the knowledge. Beside this he offers concise criticisms to the research methods and their evaluations.

He believes that, this is method which make the progression to the knowledge. appearance cannot be causes. Therefore, there is original requirements to deeper insights. there are two important kernels to this topic.

1)"coming to know means to place oneself in a conditional relation to something" (e.g. The initiative point of conditional probability).

2) there are no facts in itself but "interpretation exist as an effect(result)"(i.e. Attribution).

In more expansion, actions (i.e. events) and probabilities of them together considered in terms of all estimated ones, consequently" the properties (i.e. attribution) of a thing are effects on other things".

there is no "solid facts", but interpretation. And that stands for introduction of meaning. this implies that, "science is discoveries of importing known into unknowns. Thus, moving from familiar in to unfamiliar. This departure is prone to mistakes, due to "fictitious insertion of a subject". This can be initiative point for critical needs to statistical testing. "transforming of nature into concepts to master nature". The moving from one unfamiliar to a whole familiar.

The interpretation of an event says every change supposes an effect (i.e. Attribution). the belief in" *causaé* (efficient causes)" take us to belief in *"télé* (final causes)". According to these conspicuous keys, we have permission to explore until complete satisfaction.

Formulas is our handy gadgets to comprehend the world. Reduction a regular phenomenon to a formula make us simplification. thus there must be constant. in fact, these constant do not exist in reality then we invent them. The permanent (i.e. continuous) sequence of certain event indicates no law but a power relationship between two or more forces. Therefore, the formula is equal to "an ever recurring result "with constant consequences.

the law the "false freedom". Actually things behave "under necessity" there is no "obedience", rule or law. Thus "things are as they are".

" The question in every event is the degree of residence" (being constant/regular)" and the degree of superior power".

"Calculability exists precisely" because things are unable to be other than they are. any atom of motion effects the whole system ("butterfly effect"). Motion; "translation of effects to consequence". "Unity" does not exist but we need it therefore we make it "to reckon". "Even an atom of force concerned with its neighborhood, distant forces balance one another". It is the" kernel" of *perspectivism* [2].

Finally, that *sequential causality* together with this view we have "adjoining and concurrent dependence" (i.e. there is no absolute independence). for the ultimate point; totally the whole system of the universe is traveling toward or in the context of chaos and not in equilibrium so there is a strong atmosphere of uncertainty in every part of the universe to estimate, predict or so as a physical system. Hitherto we talked about *perspectivism;* which generally means making decision based on deep understanding simply specifying. This is philosophical kernel of this study, its methods and its evaluation and testing. You can find the philosophical roots of almost new and evolving baby of attribution of extreme climate event in perspectivism as illustrated above. I believe it works based on it, in its foundations.

Generally, when we attribute an event or events to particular results due to particular processes, we believe they are originated from those results. Actually we refer those results to their origins, in a nut shell. in mathematics the term, "attribution" and 'to attribute" is being rooted in geometry and its classification based on specific features (properties). therefore, in the case of this study or more generally, a functional relation with particular input, a rule and output [3].

2.3-Mathematical and physical foundations of chaotic dynamical systems

2.3.1-Hamiltonian chaos

The foundations of Hamiltonian chaos and statistical mechanics in particular "phase transitions", is on geometry and topology. Phase transitions are among natural phenomena. Riemannian manifolds defined the dynamics of the Hamiltonian systems. Although the major task is to recognize cause or derivation of "the chaotic instability of dynamics".

Mathematically we are dealing with "Riemannian manifolds" which are mechanical manifolds when a Hamiltonian system experiences a phase transition. Then what happened for the manifolds and how can we describes this *topological structures* and *substructures* (which are fractals in the Lorenz-63 model) in its "configuration space"?

"a phase transition can occur only at a point where the topology of these sub manifolds undergoes a transition, and this is true at least for a large class of systems".

Instability mentioned above is inherent in any dynamical system.it is dabbed as *Hamiltonian chaos* which sign the unpredictability property of the system after ascertain time interval. Then defining unpredictability of a deterministic but unstable dynamical system. *"A locally exponential magnification with time of the distance between initially close phase space trajectories is the hallmark of deterministic chaos".*

In contrast stability says that in "phase space the trajectories group into bundles and close to one another without any significant spread as time passes" so no explosion! Consequently, non-sensitive to initial condition and completely deterministic. This behavior of stability is formulated in mathematics by Picard–Lindelöf theorem (Also called Cauchy–Lipschitz theorem, or existence and uniqueness theorem) which gives a set of conditions under which an initial value problem has a unique solution. This theorem "formalizes the *deterministic nature of classical mechanics*; however, predictability stems from the *combination of determinism and stability* of the solutions of the equations of motion [132,133].

2.3.2 the role of stochastic methods

However, it might seem this title has no job with the previous title but in our factual and counterfactual theory of causality has a kernel role. Also this field is contemporary climate research and is progressively being used in wide-ranging weather and climate models.

Stochastic methods are used for parameterizations in sub grid-scale, representing model errors, quantifying uncertainty as well as data assimilation and ensemble prediction [100.10]. It is important to note that all the mentioned applications of stochastic methods in climate research are used in this study! This is an absolute need to turn the page and move on the stochastic methods since "we still cannot resolve all necessary processes and scales in comprehensive numerical weather and climate prediction models" and also this is an impossible task as the random and chaotic nature of the climate system do not allow to precisely determine it.

According to [100] reduced models are able to simulate and predict large scale –special and temporal- modes of phenomena. Based on the *Statistical mechanics and dynamical systems theory* "in reduced order models the impact of unresolved degrees of freedom can be represented by suitable combinations of deterministic and stochastic components and non-Markovian (memory) terms"[100,107,110].

According to [100] "reduction of model biases" is another advantages of Stochastically numerical weather and climate models. Although a systematic framework is needed.

"stochastic parameterizations have the potential to remedy many of the current biases in these comprehensive models" [100,10,24]. It can be because of the fact that Climate is a multi-scale system in which different physical processes act on different temporal and spatial scales [101]

2.3.3- On the stochastic processes

In contrast to deterministic processes stochastic processes have a random component. It can be said that its foundations is on the Brownian motion then the processes are driven by white noise. White noise is a serially uncorrelated time series with zero mean and finite variance [102]. To formulate and solve the related problem we must move on the SDEs house from ODEs hunt but with the requisite means! This is because a **stochastic differential equation**s i.e. SDEs are a combination of a deterministic differential equation and a stochastic process. This leads to a new calculous instead called, stochastic calculus which have no unique and exact solution as well. In explanation about this point we can say as in differential equation we also do integration but that needs to discretize and the problem will rise when "*different discretization's of its integral representation lead to different results even for the same noise realization*". According to Gardiner (1997) *Ito* and *Stratonovich* are The two most important calculi in solving stochastic differential equations. Based on [100] "the physical difference is that Ito calculus has uncorrelated noise forcing while Stratonovich allows for finite correlations between noise increments". Although switching between the twos is also possible based on the problem at hand. "SDEs describe systems in a path wise - in one path at a time- fashion". Since Fokker-Planck equation (FPE) describes how the probability distribution evolves over time [102], the SDEs and the FPE propose two different methods at looking at the same system. but the important note is that *the parameters of SDEs and their corresponding FPE are linked*; thus, one can use the FPE to estimate the parameters of the corresponding SDE [103]. This is the job which has done in this study. Although the solver is the Kalman filtering approach.According to [100,107,] Linear Inverse Models (LIM) were the first attempts at stochastic climate modelling. They were linear dynamical models with additive white noise forcing.

These linearization's of the dynamics and then add white noise and damping were due to make the models *numerically stable*. This means that the linear operator resulting from the linearization process should only have *negative eigenvalues* to guarantee stability and related

realizations. they can only produce Gaussian statistics and can be practical for instance in the case of the ENSO prediction but have no hand in the case of extremes with their high impacts. Although in current research there is a progressive effort for the "fitting of the nonlinear stochastic models to data" which are really the case on the table in recent decades [100,107.110]. Most of the traditional approaches fitted the parameters of the stochastic models without taking account of physical constraints, e.g. global stability. Many studies linearized the dynamics and then added additional damping to obtain numerically stable models. But the fact is that climate as whole and mostly its subsystem behaves in a nonlinear and chaotic fashion then these sort of merely mathematical approaches have an effect on solving the problem. However, [134] developed the nonlinear normal form of stochastic climate models and also physical constraints for parameter estimation. After that [124,9,118] among others developed the method with its physical constraints. In current studies using these physical constraints leads to successful derivation stochastic climate models along with their corresponding physically consistent.

2.3.4- what about the mathematical basis

High impacts of the rare climate events makes them very crucial to study. Then the current scenarios of the climate change might have any usage in this context of study. Instead the most least thing we can do is to "gauge the risk" because it is impossible to avoid such high-risk scenarios. This would be by assessing **their probability of occurrence**. Although because of the supposed autonomous dynamical system in this context the time dimensions are important as well [24,124]. Traditionally and generally in this context χ represents the space of realizations of a time series or stochastic process of random variables of interest, say X0, X1…, which represents as a "theoretical model for treating time series of quantities of relevance for the phenomena under investigation".

This probability space χ equipped with a σ-algebra β of events and a probability measure P planned to quantify the probability of the occurrence of any event A ∈ β. Consequently, we can characterize *rare events as those events $A \in \beta$ such that $\mathbb{P}(A)$ is small.* Fundamentals of the analysis of the properties of rare events begins with the study of EVLs. *However, the main innovation presented here is the study of rare events in the context of dynamical systems. Although it is stated earlier that, it is usual to use EVT to study the properties of a time series representing the time evolution of a random variable. In fact, you can bring together* ***randomness*** *and* ***time evolution*** *by considering purely deterministic systems featuring a*

chaotic dynamical system. Actually the research field focusing on the study of how systems evolve with time is usually referred to as dynamical systems**.** [69, 70,110]. "*In the last century, the discovery of the ubiquity of chaotic systems and the investigation of their erratic yet extremely structured behavior has shed new light on the interpretation of stochastic processes*".

A dynamical system consisted of a ***phase space***, where each point defines a given state of the system, ***a time evolution model***, which can be continuous or discrete, and an ***evolution rule*** that drive the unfolding of the system. Although in the continuous case, the evolution laws are usually given in terms of ***differential equations***, while in the discrete case **a map** describes the transitions from one state to another. One of the most famous examples of a dynamical system is the celebrated Lorenz '63 model [105]. It consists of a system of three differential equations designed to provide a minimal representation of convection via a minimal representation of its dynamics. This system provides the prototype of chaotic behavior and has been extensively studied in a variety of disciplines including mathematics, physics, geophysics, and time series analysis.The erratic behavior of this system makes its outputs hardly distinguishable from purely randomly generated numbers, at least in the long run. This led to the coining of the expression butterfly effect, or, in more precise terms, the sensitivity to initial conditions and the presence of impassable and intrinsic limits to the deterministic prediction for a chaotic system.

It is introduced some terminologies for dynamical systems, taking the example of discrete time evolution. In this setting, χ denotes the phase space for which we endow some topological, or differentiable, or measure-theoretical structure, depending on the problem under consideration. According to above and denote by β the associated σ-algebra of subsets of χ, which gives the measurable structure. The system itself is represented by a measurable map (with a structure compatible with that of β) that we denote by T (or sometimes f) such that $T : \chi \to \chi$ describes the time evolution of the system, that is, determines the rule defining the transition from an initial state $x \in \chi$ to the state $T(x)$, after one unit of time. Moreover, we require the existence of a probability measure $\mathbb{P}$ defined on β that is coherent with T, in the sense that, $\mathbb{P}\left(T^{-1} \quad (A)\right) = \mathbb{P}(A). \forall A \in \beta$, which we express by saying that $\mathbb{P}$ is T-invariant [24,110,107].For an initial state $x \in \chi$ we define its orbit to be the sequence of states through which the system will go if it is started at that particular state x, that is, the sequence x, T(x), $T^2(x)$…, where

$$T^n(x) = \underbrace{T \circ T \circ \ \dots \ \circ T}_{n \text{ times}}(x)$$

is the nth iteration of T. *The main goal of the dynamical systems theory is to study the long-term behavior of the orbits of the system*. If this can be reached in probabilistic context, then th fractal structures define and display this iteration.

As in the case of defining an event for a sample space the rare event is also given by some subset $A \subset \chi$ of the phase space belonging to β and such that $\mathbb{P}(A)$ is small. **The occurrence of a rare event is obtained whenever the orbit of some point hits the target set A.** Poincare's Recurrence Theorem guarantees that almost every point of A will returns to an often infinite state [106,107]). Moreover, if a system is ergodic with respect to $\mathbb{P}$ and $\mathbb{P}(A) > 0$, then the orbit will hit A at some point in time. *A system is said to be ergodic with respect to $\mathbb{P}$ if for every $B \in \beta$ such that $T^{-1}(B) = B$ then either $\mathbb{P}(B) = 0$ or $\mathbb{P}(B) = 1$.* In other words, a system is ergodic if it is not decomposable into two parts (detected by the measure $\mathbb{P}$) that do not interact with each other, [106,17,24].

2.4- On detection and Attribution of extreme events

Attribution; according to (Hegerl et al,2010) the method of assessing the "**relative contribution" (probability) of various causal factors to a change or event with an "assignment of statistical confidence**"[4].

Extreme event is a rare climate or weather event in a region. It statistically is, **departure from expected means.**

"As the evidence for anthropogenic climate change continues to strengthen and 6 concerns about severe weather events increase, scientific interest is rapidly 7 shifting from detection and attribution of global climate change to prediction 8 of its impacts at the regional scale"[77].

We can attribute an extreme event to its dynamical (physical) roots, merely based on a robust, rigorous and optimal mathematical theory of **rules of the game** for decision making. That is, the climate provides a chaotic, interdependent, cooperated and dynamical system. Therefore, any phenomenon in this system, is the outcomes of their inhomogeneous players (i.e. subsystems). These rules assistant us for differing extremes from regular events [24]. They generate "easily adaptable conceptual framework", to deliver well limited universal properties

for more robust statistical inference. This is a very need in the time of interpretation of data (observational and simulated).

For more certain estimates to high quantiles(limits) and occurrence probability, we need particular arguments. These arguments define us distribution tails properties according to general and initial conditions of generating function.

According to a classical and intuitive view, the mathematical basis of extreme value laws is; **the more intensive the more obscure; i.e. the more extremes the less probable** [24]**.**

In calculous the maxima or minima of a **function** dabbed as **extreme values.** in addition, we have a point between this two extreme limits and that is called critical point which actually is the transitory point of two upper and lower limits. For a series, these limits represent the initial and terminal terms. Then the **distribution of these extreme values** are called the extreme value distribution. For accurate and precise evaluation of this function we need optimization methods [3].

There are three common different approaches for detection and attribution of climate change studies. According to [25] they include Fingerprint-Based Methods, Non-Fingerprint Based Methods, Multistep and attribution without detection methods and methods for extreme event attribution. As noted before in attribution methods such as fingerprint methods the assumption is that natural variability and anthropogenic forcing are constantly playing their role while the fact is that according to [26] " attributing an event solely to either human-induced climate change or natural variability can be misleading when both are invariably in play". Based on [27] there exist two main approaches for extreme event attribution. The "risk-based approach" - estimating how the probability of event occurrence correlates with climate change- and the "storyline approach" - evaluating the influence of climate change on thermodynamic processes leading to the event. According to [28] with the currents event attribution methodologies, we cannot "conclusively" attribute the recent decade's extreme weather and climate events to the "anthropogenic forcing". Based on this study the most critical point in any attribution study is the initial framework. That is we can have different inference and different decision making about a particular extreme event. "What we have now are multiple climate models that allow us to cut down on the noise" (Peter Stott) [29]. In [27] Trenberth and et al, concluded that almost all of attribution study is on the model simulation results with their almost high level of uncertainty and most importantly their lack of " physical credibility" which can make the decision making process more efficient. They declared that " The conventional attribution

framework struggles with dynamically-driven extremes because of the small signal-to-noise and often uncertain nature of the forced changes". Therefore, for unfolding the reason of recent extreme events especially rare ones we ask " why such extremes unfold the way they do?". The answer as noted earlier turns back to the initial notion to the problem. Therefore" it is more useful to regard the extreme circulation regime or weather event as being largely unaffected by climate change", and ask "whether known changes in the climate system's thermodynamic state affected the impact of the particular event". The certainty about the dynamics of the atmosphere is obviously dubious and confident "reliable statements" about the role of climate change, can be made "mainly through the thermodynamics". The robust basis is on the consideration of temperature fluctuations. In fact ΔT controles ΔQ as noted earlier. Although based on [31] "understanding how the overall risks of extreme events are changing in a warming world requires both a thermodynamic perspective and an understanding of changes in the atmospheric circulation "and "Every extreme weather event is ultimately caused by a unique combination of external drivers and internal chaotic variability". Despite the utmost linear response of thermodynamic consideration of the climate system the dynamic response can often be non-linear and may have completely contradictory result. In fact, "the dynamic contribution is often of a similar magnitude to the thermodynamic response". Therefore, they accompanying together.

The thermodynamics of phase change/transition of moisture in the atmosphere consider its vapor, liquid and solid state through the Clausius-Clapeyron equation. That conventionally is based on P-T diagram which indicate "equilibrium vapor pressure/saturation vapor pressure(SVP)". Actually it is curve there that Clausius-Clapeyron equation yields its slope.

$$\frac{dp}{dT} = \frac{\delta s}{\delta V} = \frac{L}{T\delta V} \qquad (5.2)$$

Where δs , is gained positive **entropy**. δV is positive volume changes. L is latent heat. The note is that this is the latent heat of evaporation per unit mass which also dabbed as specific **enthalpy** of evaporation and given by $L = T\delta S$ [18].

According to this equation saturation capacity of the atmosphere has a growing exponential distribution at a rate of about 7% C^{-1}[26]. In brief, according to the present knowledge in extreme event attribution and also attribution by its own, and base on [26]

a fruitful and robust approach to climate extreme-event attribution is to regard the circulation regime or weather event as a **conditional state** (whose change in likelihood is not assessed),

and ask whether the impact of the particular event was affected by known changes in the climate system's thermodynamic state (e.g., sea level, sea surface temperature, or atmospheric moisture content), concerning which there is a reasonably high level of confidence".

Of course this leads to a "physically-based approach". These physical process is strongly related to the event in interest. In spite of their difference "from the conventional frequentist" method, "**they are perfectly reasonable from a Bayesian perspective"**. The note is that; different proposition of the interested questions leads to different meanings of the answer. Additionally, this method more emphasizes on impacts, thus able to yield more convenient information.

According to [27] the question is that "was this event influenced by climate change?". The response to this question goes through these steps:

1- The definition of the event of interest
2- The way of defining causation.
3- Defining natural variability ("a world without climate change")

A multistep method is grounded on climate conditions. These conditions are meticulously associated to the variable or event of interest. In this approach, "an observed change in the variable of interest is attributed to a change in climate or other environmental conditions". Consequently, "the changes in the climate or environmental conditions are separately attributed to an external forcing, such as anthropogenic emissions of greenhouse gases". For higher levels of confidence, we necessity need an in-depth understanding of associated physical processes. According to the different situations, this method can be use with or without detection [25]. The rigorous theory of causality is the main theoretical framework for any attribution study [38]. As noted before it is stemmed in the works of great philosopher such as Nietzsche. Which I dare to say that whole of this work is based on his fabulous ideas.

Also there are different methods in attribution of extreme events based on risk assessment, statistical analysis for time series and trends, and physical perception. Among different approach for the purpose of this study like in the case of attribution we choose "conditional or ingredients-based attribution" which generally is practical for both [25].

Lozano et al (2009) published an initiative article which was almost totally different from the current issues of event and extreme event attribution articles. They use "Spatial-temporal Causal Modeling" instead of common only model simulation dependent approach. It was a "data centric approach". it was depended on "actual measurements of climate observations and

human and natural forcing factors. They applied their method on the two series of monthly and yearly data from NOAA 2.5×2.5 data archive in a period of 1990-2002 using spatial statistics method like "thin plate spline".

According to [27] the first step in extreme attribution study is the framework and second is the exact definition of the event in interest. In the case of this study we used conditional framework and then defined our interested event mathematically then physically.

The conditional method has been proposed to investigate the " Influence of climate change" on particular extreme event. There are "different nuances(degrees) of conditioning" to all the forcing leads to climate change. For example, conditional to SST/SIC, sea level rise, a particular pattern of circulation, El Ni ˜no/La Ni ˜no, anthropogenic effects [33].

In a study based on unconditional attribution, man-made climate change directly attribute to an extreme event or the impacts of it. You can find, this sort of attribution methods in studies which merely depend on, observations or coupled models (e.g. CMIP5, Lorenz 63 or others). They usually compare pre-industrial (i.e. natural-forcing) and historical runs/model runs. Therefore, their conditionality is only relying on the models' capacity [33,34].

According to [27] conditional attribution methods relates man-made climate change joint to a "precursor"(originator) to both of extreme event, or its impacts. This originator is an internal component stemmed in coordination of climate system. based on conditional attribution method this precursor plays a title role in the event occurrence. Then the method combines thermodynamic originator with the anthropogenic factors such as GHG and aerosols [35].

A thermodynamic precursor is an originator that is directly proportional to the temperature increase. Consequently, then conspicuously influence the climate change. SST is the most useful one [32]. Computational costs of coupled models, the alternative could be atmospheric models. The boundary condition in these sorts of models is based on SST variations. They assume better representation to dynamics processes and interactions between surface and atmosphere (thermodynamic process) [36]. In this context, **the higher the resolution the higher the reliability**. In [36] researcher use null hypothesis assumption for their SST conditioning method. Through thermodynamics the conditional approach is able to links climate change to an uncertain dynamical originator by "storyline method"[37,38]. Actually the more dynamic dependent events the more internal variability climate change signal. Therefore, how does the climate change impacts on atmospheric circulation? or What is its best contribution estimate of it to the observed event [27,37].

"Causal counterfactual theory provides clear semantics and sound logic for causal reasoning and may help foster research on, and clarify dissemination of, weather and climate-related event attribution"[1].

Bestowing from [30], in probability theory, events define as "All the possible outcomes of an experiment that form the fundamental probability set {A}". they include certain event(I), the impossible event(O), random events and **elementary events/**(E). The last one are denoted to "all possible outcomes of the experiment excluding each other" therefore they are not **compound events**. the operation properties of "the fundamental probability set forms a Boolean algebra with complement, addition, and multiplication" which is dabbed as "field of events". According to this another important notation is "the complete system of events(A_α)". Which $\alpha \in \theta$, and θ a finite or infinite set of indices. This membrane is allowable if $A_\alpha\, A_\beta = O \; for \; \alpha \neq \beta$ and $\sum_{\alpha\in\theta} A_\alpha = I$.

If $A \subseteq B$ then the occurrence of A follows the occurrence of B. The elementary/simple event can be define as $E = A + B \; then \; E = A \,.\, E = B$, and if doesn't satisfy this condition is the compound event. If $AB = O$ then exclusive or disjoint event. The A and B cannot simultaneously occur.

In many detection and attribution methods we seeking for an optimal value which is the best possible value (maxima and minima) of a distribution and the solution leads to this is optimal solution. Mathematically, if "in a matrix M, E (x_i, x_j) is the expectation, where x_i and x_j are mixed strategies then, according to Springer Handbook of Mathematics, (Fifth Edition ,2007) Definition of the Extreme Point is:

' A point $\underline{x} \in M$ is called an extreme point or vertex of M, if

$\forall\, x_i \,.\, \exists\; x_j \in M$ with $x_i \neq x_j :\; x_i \neq \lambda x_j + (1-\lambda) x_j$.

$0 < \lambda < 1$. i.e. x is not on any line section linking two different points of M". so our task is delving for this limit.

$$\lim_{n\to\infty} M_n$$

For a statistical solution in its frequentist view, we have a statistical model defined by three general parameters (K, ζ, β).

The primarily calculation is fitting for the empirical probability of block maxima distribution (BM). It is originated from splitting a time series of lengths, (e.g. number of precipitation days) into blocks (m). Then investigate maximum value of per blocks. This found Generalized Extreme Value (GEV) distribution. In fact, this distribution is generalization of the Freshet, Gumbel, and Weibull distributions in studying of extreme events. In GEV after blocking by BM method we need to define the shape parameter(K) for defining the qualitative properties of the extremes. The positive(K) provide Frechét distribution, that means the extremes are unlimited. The negative (K) gives Weibull distribution, which means the extremes are upper limit (i.e. Mode). The neither positive or negative (k) offers Gumbel distribution which means critical point, therefore, vanishing shape parameter (i.e. mean).

The location (shift) parameter(ζ), pronounces magnitude of the extremes. The scale parameter (β) designates the extremes variability. GEV is based on BMs but there is another method based on correlation (i.e. a one-to-one relation) between BMs and thresholds according to initial conditions. In fact, the PGD is based on POT.

A decisive side is that, from an opposite view but under the same mathematical conditions for defining limit laws. "One finds a one-to-one correspondence between the statistical properties of the BM in the limit of very large n, and those of the *above-threshold* events". Under the conditional probability of X, X hits a value of definite threshold T, for very large T. in this states if $T \rightarrow \infty$ and if the X distribution is not upper-bounded , or, $T \rightarrow max\, p(x)$. we can show that distributions these maxima are based on the two-parameter **generalized Pareto distribution** (GPD) [1, 40, 41].

this way of looking at extremes leads to *peaks over threshold* (POT) method. We use this method in the time of passing the point of high risks. Recognition of risk value can be solved by fitting data with a GPD. It is also common to calculate its Value at Risk (VaR) and Expected Shortfall (ES) fitting data with a Generalized Pareto Distribution (GPD).

At what time of POT statistical inference, we have, the *shape* and *scale* parameters in GPD, analogous to GEV case [43]. hence if we know the properties of defined BMs we can deduce the POTs properties. Also it is possible to find a one-to-one between β and ζ parameters of GEV and Κ and T of GPD, such that this defines a one-to-one relation between the two distributions [44].

2.5- Physics of the climate and currents models

we debate here the basic physics that controls the Earth's present climate, with certain highlighting the **energy and water cycles.** This desires a number of **"dimensionless parameters",** relevant to general circulation depiction of the planet, its climate, and hydrological cycle. preceding climate different phases are meters of future climate change. The atmosphere merely is one portion of the **"parameter space of factors"** of the weather and climate.

The initiative point is **Greenhouse gases** along with **radiative forcing** associated with various drivers which are parts of the earth's climate. According to definitions of short and long waves radiations they are the most important points to consider. Both of them have their particular effects on the climate. Lorenz in 1955 put away the mechanisms of energy forcing, conversion and dissipation and their relation to the general circulation of the atmosphere. Based on this basis **Climate is a forced-dissipative thermodynamic system originally** [18,68].

Either the atmosphere and ocean are systems which are out of equilibrium. They "**irreversibly exchange** matter and energy from their environment and re-export it in a more degraded form at higher **entropy**". low-entropy solar photons emitted at 6000 k, high-entropy thermal photons emitted at 255 k. special gradient in chemical concentration and temperature also their related matter and **energy fluxes** can be stablished and maintained for long time within non-equilibrium system. Like large scale atmospheric and oceanic circulation. **Greenhouse effec**t, by which "radiation-absorbing gases in the atmosphere raise the surface temperature above the value that would occur if they were not present"[18,68].

In fact, the so-called solar constant or solar irradiance **(S_0 = 1360.8 W/m^2** at solar minimum) fluctuates **by ~0.1 Wm^{-2} in excess of 11-year solar cycle**. The fact is that, this is a negligible deviation for the climate of the earth. In the normal radiation budget of the Earth the absorbed SW flux equals the emitted longwave LW flux. This is dabbed as equilibrium state which means;

$$\frac{S_0(1-A)}{4d^2} = \sigma T_e^4 \tag{6.2}$$

where A = 0.293 is the global albedo, d the Sun-earth astronomical distance - d=1 for Earth- σ is the Stefan-Boltzmann constant, and T_e is blackbody effective temperature which yields the observed LW flux. But now due to man-made emissions of greenhouse gases and other climate forcing, the Earth has lost her equilibrium by ~0.6 Wm^{-2} [41].

TOA influences to the global energy balance, implemented through the atmospheric column. Virtually 1/2 of Earth's planetary albedo sources is her clouds, while; the other 1/2 is from the clear atmosphere. Aerosol, gas and surface have practically equal scattering. Then the contributions of the atmospheric LW outflux is ~92 %, and the remaining of it is ~8 % which belongs to the surface that losses through the thermal infrared window. However, in clear skies, it reaches to ~25% owing to the LW opaqueness of greenhouse gases [42].

The peaks of solar radiation is placed in the visible spectrum. mainly ~22 % of it is absorbed. O_2 and O_3 in the ultraviolet, H_2O in the near-infrared, **and absorbing aerosols** in the visible. primarily the absorbed SW influx that touches the surface is about 88%. **The balance of surface energy budget is largely on the upward turbulent motions, latent heat, heat fluxes and somewhat on the net (up–down) LW outflux** [43].

T_e = 255 K, 33 K colder than Earth's surface T_s = 288 K. This difference is a measure of Earth's greenhouse effect [44] due to LW absorbers. **According to [45] about 50 % of the greenhouse effect is due to Water vapor, tracked by clouds ~ 25 %, CO_2 ~ 19%, other gases and particles just about ~7%.** Actually the primary role for driving the Earth's climate is on the Carbon dioxide. Nevertheless, this is a conjoint role with other drivers. Although by the Clausius-Clapeyron equation, water vapor is controlled by the heating role of CO2 and other non-condensable greenhouse gases [46].

Based on [47] the cloud forcing (cloud radiative effect) stands for the TOA difference among absorbed SW in-flux and LW out-flux in clear sky, plus the total fluxes in all conditions. However, **SW** in-flux is controlled by the variation of solar zenith angle with latitude -in the clear sky-, but its zonal mean at each latitude indicate details of important radiative role of Earth's surface [14]. This role is least over bright ice-covered Greenland and Antarctica and to a lesser magnitude over the Arctic and Antarctic sea ice packs. The supreme **role is over the dark oceans**, and the middle role is over other land surfaces.

Other fluxes are controlled largely by the atmosphere of the earth. Consequently, these are meters for the general circulation. Dynamics of climate regimes in tropical and extra tropical regions controls by the radius, rotation period, and thermodynamic structure [43].

Low-level of moisture convergence and deep convection in the rising branch of the Hadley cell makes the most of the greenhouse effect near the equator then produce a minimum LW out-flux to space in clear sky, relative to the flux maxima in the surrounding subtropical dry subsidence regions [14]. Otherwise the clear sky out-flux primarily leads to the decrease in

temperature and water vapor with latitude. Longwave positive cloud forcing maximizes in accordance with the tropical locations of deep convection. Also another maximum exist in the extratropical storm tracks. However shortwave negative cloud forcing also is large in magnitude in these regions. Additional SW negative cloud forcing maxima also exist off the west coasts of the continents though with no corresponding LW signal because of stratocumulus decks there [50]. In averaged, SW cooling by clouds exceeds LW warming by ~17 W/m^2 in global scale. This means that the net effect of clouds leads to cooling for the current climate of the planet [48].

According to current knowledge Different effects of **Aerosols** could be consist of: Direct, indirect and semi-indirect effects.

The direct effects are due to their interaction with SW radiation. Because the most aerosol types such as sea salt, sulfates, nitrates, , and organic carbon are bright, they cool Earth. Among them black carbon and mineral dust absorb SW radiation. Although this cool down the surface by precluding reaching sunlight but warms up the atmosphere and also the whole planet.

Indirect effects are due to their interaction with clouds. Aerosols are cloud condensation nuclei thus affect the number and size of cloud droplets. Therefore, the increase aerosols, the more available sharing water. However smaller droplets, increasing their $(\frac{A}{V})$ ratio and thus making the cloud more reflective consequently cause cooling.

Semi-direct effects are due to the influence of absorbing aerosols warming. This affects atmospheric thermodynamic structure. As a relevant example, under strong inversions marine stratocumulus clouds develop. Its extension may increase if an adjacent continental advection of absorbing aerosol surges the inversion strength [45].

According to [49] the geographic distribution of aerosols is determined by natural and anthropogenic sources, tropospheric chemical reactions, wet and dry deposition, and also as noted above, advection by the general circulation. Sulfate and nitrate are chiefly have anthropogenic origin as they are adjacently concentrated near the industrial, transportation, and agricultural regions [50]. The most prevalent aerosols in the biomass burning regions are black carbons. nevertheless, they also belong to the main industrial and transportation regions. In the case of mineral dusts, they mostly have natural origin. Sahara and Arabian deserts are the largest source. However current man-made desertification process is exacerbating the natural phenomenon. The main source for producing sea salt is the oceanic regions of strong surface

winds such as the extratropical storm tracks. Generally, global balance of SW and LW fluxes of the TOA, leads climate equilibrium. Nevertheless, this cannot be true locally, because of imbalances in latitudinal variation of net radiative heating [14].

The receiving and emitting differences of sun light between the tropics and higher latitudes and the role of the oceans for turning back the equilibrium to the entire system are the kernels for driving the circulations in the atmosphere and ocean [51]. The initiative points for heat transport in the atmosphere are the Hadley cell and synoptic-scale eddies which are rooted in the bar clinic instability of extra tropics and high latitudes [43]. At low latitudes the ocean circulation dominates this transport.

Two essential mechanisms of the oceans according to [14] are

1- the first ~100 and surface circulation by the surface winds friction and also the Coriolis force.

2- a thermohaline circulation by density gradients due to differences between temperature and salinity.

Actually this is the surface winds which yield, eastward ocean currents in the tropics and westwards in the extratropical regions. Deflection of the easterlies to the poles and the westerlies to the equator are the causations of, ocean gyres. they transport warm water to the poles, off the east coasts of the continents, and cold water to the equator, off the west coasts. By this process, **gyres complete a net heat transport to the poles** [50]. If you remove or negatively change this process, you will encounter with another ice age as the climate cannot response to the external forcing.

The conspicuous sliding slow Vertical movements in the waters of the equators and higher latitudes denotes that the effects of variations in greenhouse gas concentrations will be revealed in changes of surface temperature for the long term of proceeding centuries (14). Vertical heat transport in the ocean is also an important basis for the equilibrium state of Earth's climate. The radiative fluxes and laps rates are the dominant drivers here. *in a theoretical vertical profile in equilibrium, divergence of the radiative flux is zero at all levels*.

In this respect the equation of

$$T_s^4 = (1 + c\tau\ )T_e^4 \qquad (7.2)$$

Shows the opaqueness of the earth's atmosphere. where the $\boldsymbol{\tau}$ is the LW optical thickness, **(c)** is a constant and the subscribes of **(s)** and (e) denotes to the surface and atmosphere respectively. this is a primary equation. therefore, surface temperature exceeds its **effective temperature,** with the optically thicker –greater $\boldsymbol{\tau}$ -atmosphere. Generally, it depicts the basic physics of how the **radiative fluxes of the atmosphere in vertical dimension controls surface temperature [52].**

Models based on this equation essentially yield discontinuity between the temperature of the surface and atmosphere. Because the surface absorbs all the incident SW radiation (based on the equation). Moreover, the predicted T_s significantly go beyond the observed T_s for realistic τ. However lower troposphere lapse rate of Earth in radiative equilibrium is **super-adiabatic** [53]. Consequently, a correction, either to a specified threshold lapse rate (dry or moist adiabatic) or defined by a more thorough model of **convection**, is used to bound lapse rates and yields a true T_s and temperature profile. According to [53] this upward heat transport by convection decreases the sensitivity of T_s to fluctuations in atmospheric composition or insolation.

The laps rate in the tropical regions is up to ~400 hPa. The moist adiabatic convection is the controlling process. This process over this level is irregular because of entrainment of dry air which limits this process [54]. This happens because the sufficient humid boundary layer generates frequent moist convection to regulate conditionally unstable lapse rates to near-neutral stability. In cooler regions of the tropical ocean, the weak Coriolis force allows advection to regulate the free tropospheric lapse rate to a closed value to that in the warmer ocean regions [48]. however, over the land in the tropics, the lapse rate and convection is unstable and quick due to less solar heating and available moisture [14]

Stability in the extratropical regions is static which indicated that synoptic transports of baroclinic eddies and moist convection affect extratropical lapse [55]. The subtropics convergence and divergence are the drivers of Equilibrium in the water cycle (i.e. The balance between P and E). P > E is obvious in rising branch of the Hadley cell and the extratropical storm tracks. E > P is evident in the dry subtropical descending branches of the Hadley cell [50]. Here net moisture convergence into the equatorial region (ITCZ) and net moisture divergence from the subtropics brings the equilibrium again. **Hence major extent of tropical precipitation does not originate from locally evaporated water.**

The tropical and extratropical dynamical regimes in addition to upwelling and down-welling regions make four basic climate zones [14,50].

a) equatorial: branded with weak stratification, high humidity, and regular strong precipitation.

b) subtropical: indicated by weak stratification, low humidity, and thus weak precipitation.

c) multitude: depicted with stronger stratification and high humidity, and consequently regular and moderate precipitation. d) polar: described by tough stratification, low humidity, and in consequence weak precipitation.

The above equation shows a purely radiative energy cycle at TOA. Conversely, the energy cycle and water cycle at the surface and within the atmosphere, are well mixed. Turbulent fluxes of sea surface, equilibrates most of the absorbed sunlight. This occurs by the latent heat flux (evaporation). Therefore, the ratio of sensible heat to latent heat flux is small. But according to soil moisture over land this ratio is significantly different [14,50].

2.5.1- Climate sensitivity to external forcing and climate models

inequity between absorbed SW radiation (Q) and outgoing LW radiation (F) is the single means for the global climate change. Based on equation (1), this can only be started by a change in solar radiance. consequently, a forced change in planetary albedo A (due to a volcanic eruption, aerosol emissions, deforestation, desertification etc.); or a change in greenhouse gas concentrations (which changes the LW opacity and thus moves the emission-to-space level to a different altitude, temporarily changing T_e).

When **external forced climate change** occurs, internal feedbacks modify A and T_e as the system shots to equilibrium. In the **slow climate perturbation** due to anthropogenic emission of greenhouse gases, the most immediate reaction is, cooling of the stratosphere, which is crudely in radiative equilibrium. Consequently, warming of troposphere then land surface happens shortly after that [52]. **These processes lead to increment in height and emission level of the tropopause**. However, warmed troposphere, that positioned the T_e at the new emission level is not very deviated from the level before the commencement of the climate change. Hence, **present climate is merely out of equilibrium by ~0.6 W/m²**. In an ongoing continuous process, "convective adjustment" also, increase the surface temperature. Actually this is stemmed in the existing deeper troposphere. for the reason that of the thermal inertia and heat capacity of the ocean and climate feedbacks [14,50].

Generally, “GCMs estimate equilibrium in climate sensitivity to external and internal forcing by immediate doubling of the CO^2 combined with Q-flux of the ocean. consequently, F reduces since the opaquer atmosphere, leads to an imbalance Q > F and generate warmer planet. In the time of balance, Q = F again. This resulting surface temperature perturbation (ΔT_s) is called **equilibrium climate sensitivity** [18,50].

One of the promising ways to reinstruct climate sensitivity is to use Earth’s past climate changes. Earth has undergone various glacial and interglacial epochs. Based on the common theories, more histrionic climate changes happened in the distant past. For example, the hypothetical snowball Earth periods. Episodes of Neoproterozoic Era, or the paleoclimate signals for these distant epochs. More recent eras like, mid-Holocene are recognized better. These eras include trivial climate changes (up to $1C^o$). this triviality makes them less useful for future insights [56,57]. The period of Last Glacial Maximum (21 ka) is the best and largest magnitude in documentation for the climate change. However, the most accepted assumption is the Milinkovic cycles for the variations in orbital movements of the earth around sun (i.e. the fluctuation in orbital eccentricity (e) [14,50].

According to this, the LGM and the earlier glacial and interglacial epochs can be used to conclude the future climate sensitivity. However, the LGM climate is less recognized than is needed. But some attempts have done to simulate. Then most of them use proxies, such as (SST) anomalies derived from ocean proxies which simulate the LGM with GCMs used by the CLIMAP Project which use multiple proxies [58,59]. In the earlier the SSTs of CLIMAP had some uncommon features such as high tropical cooling. Consequently, implied a low climate sensitivity, and multiple GCMs could not reproduce them. In the former LGM refurbishments, discover larger tropical cooling that suggests a higher climate sensitivity.

In recent years GCMs, run coupled atmosphere-ocean models then use the reconstructions for evaluation. However, the problem here is that, they generate more or less conspicuous discrepancies among the models and the data, consequently the proxy's interpretation is actually unpractical [60].

Then the usage of LGM as a proxy for future climate sensitivity in the GCMs deliver contradictory results. Such as identical climate sensitivities in response to LGM climate forcing, after doubling the concentration of CO^2 . These inconsistencies were mainly attributed to cloud feedbacks that flip the signals for colder and warmer climates. Then they offer a nonlinear climate system. convective available potential energy varies differently in response

to +2 and –2°C SST changes because of imbalance between convective heating and radiative cooling [61].

Hitherto, we found that past climates propose limits for the future climate, but for a more convincing result it is better to , separate responses based on different type of examined climate change or do this for uncertainties in the their own observations[14,50,57] .

according to [62] twentieth century global warming is the most common “metric” for more realistic consequences in climate models. However older GCMs normally have been well matched with the global warming of twentieth century. They do this even though there exist a wide range of climate forcing and sensitivities within the observational uncertainties. That is the forcing and sensitivities display a negative correlation [63]. However, "CMIP5 generation of models does not show this behavior", because **these models try to involve aerosol indirect effect on clouds** [64]. This may cause the departure of observed trends.

a model with more aerosol cooling might also shows a higher sensitivity to CO2. Although in the current GCMs, such as the fifth Coupled Model Inter-comparison Project (CMIP5), estimations of climate sensitivity are running in different approaches. The (CMIP5) employs regression, depending on the temporary climate change of a wholly coupled atmosphere-ocean model [62].

Lack of quality in twentieth century observations causes uncertain combination of forcing and feedback, and that causes temperature record uncertainty. Therefore, we encounter with problems in validating the climate models. Another problem is that no GCM, could not make a duplication from the observed warming just with natural climate forcing. Subsequently we can say that anthropogenic influences cause global warming in recent decades [65].

In this sense today’s climate models are "wrong but useful" [45]. Chaotic nature of the climate system makes it sensitive to errors in the initial conditions and the parameterization of unsolved processes like moist convection. Therefore, **it needs probabilistic analysis [66]**. The central role for 20th and 21th century climate modeling is on the position of the tropopause and deep ocean heat capacity. But decadal variations make long-term simulation problematic. Genetically Uncertainty in the climate system because of its chaotic nature similarly leads to uncertainty in its physical models [66].

GCMs have projected future climate change for four decades, with slight reducing of uncertainty of ΔT_s in result of doubling CO^2 concentration. This increase the surface temperature by ~1.2°C, while much of the climate models, yields ~1.5° to 4.5°C of ΔT_s after doubling the CO^2 due to different negative and positive feedbacks of forced CO^2 climate change [14,44].

2.5.2- Climate feedbacks and climate models

the principal feedbacks and their signals are the **Planck feedback**. For a warmer atmosphere, the **greater emission feedback** is negative while **water vapor feedback** due to the greenhouse effect is positive. **the moist adiabatic lapse rate feedback** due to temperature difference between the surface and higher altitudes is negative. **the snow/ice-albedo feedback** is positive because of planetary albedo lessening due to melting snow and sea ice as climate warms. **the cloud feedback** also is positive due to changes in its cover, height, and optical thickness [14,50]. According to the Planck feedbacks there exist numerical differences among various models but **undoubtedly they have true signals.** Most of the deviations in ΔT_s roots in uncertainty about the **signals and magnitude** of the cloud feedback.

However, there is bilateral judgments about the mentioned feedbacks too. For instance, in the case of cloud feedback the dominant claim is that global warming cause more water evaporation from the oceans and this generate more clouds that, increase sunlight reflection and cooling the atmosphere. hence it depicts as a negative feedback. Whereas based on Clausius-Clapeyron equation the **mixing ratio of water vapor**, fluctuates intensely with temperature, while cloud formation does not depend on it. But it depends on the relative humidity, static stability, and other factors based on environment. These factors do not show conspicuous variation against warming. **Therefore, the cloud increment or decrement is subject to question.** Moreover, different altitudes of clouds, in their vertical position, leads to opposite signal because of different radiative effects. The low clouds of stratocumulus, considerably reflect sunlight however with little effect on outgoing LW radiation, leads to planet cooling. Therefore, with their increment or decrement we have a negative or positive feedback. The thinner Cirrus clouds, conversely, reflect a smaller amount of sunlight. But their position of emission level is above the clear sky. Consequently, they lessen LW emission to space. Therefore, their increment or decrement would cause positive or negative feedbacks. The cases mentioned above indicated that based on cloud types, Changes in cloud level and optical thickness fluctuate the cloud feedback.

The method of cloud forcing for calculating feedback is biased [67]. Therefore, the cloud radiative effects are differed against the effects of clear sky radiative fluxes. Consequently, they proposed "radiative kernels" for cloud feedback calculation. They used, partial derivatives of TOA fluxes based on different parameters from different feedbacks. Using this method, they achieved a more accurate depiction of cloud feedbacks. According to this results all the CMIP3 models have both a near-neutral and positive cloud feedback. Moreover, the range of climate sensitivities for the CMIP3 was 2.1°C to 4.4°C. Primarily signs from the GCMs of the CMIP5 did not revealed dramatic changes to the previous situation. However, according to [68] there have been generated advances in understanding for the net positive cloud feedback sources.

The net positive cloud feedback is a positive LW cloud feedback because of an upward shift in the altitude of high clouds level, and a positive SW feedback due to a decrease in total cloudiness outside the polar regions [14].

The first as a robust result depends on intensified convective processes as the surface temperature increase. The second depends on, two coupled effects: **the expansion of Hadley cell** leads to a shift of the storm tracks to the poles. This robust result in combination with low-latitude stratocumulus and shallow cumulus cloud cover decrement. At high latitudes, warming increases cloud optical thickness, and this is a negative feedback.

In brief we can say that the response of climate model to clouds cover perturbation because of global warming is an upward and outward shift of clouds cover and level of latitudes from the tropics to the higher latitudes [66].

2.6- Efficiency and practicality of currents climate models

The CMIP5 models are ensembles of different models with different structures and parametrizations. Therefore, it can yield reasonable results. An ensemble of different parameters can be more reliable method. Because this approach uses separate parameters in single model for simulation. Parameters can adjust to different simulation over their uncertainty range. Consequently, the compression of different effects of different parameters on the sensitivity of model is doable. The apparent dominance of positive feedbacks on the climate system, probably is a herald for future **exponentially continuous warming**.

However, it is essential to consider that:

Based on skill estimates from hind casts made over the last couple of decades, recent studies have suggested that considerable success has been achieved in forecasting winter climate anomalies over the Euro-Atlantic area using current-generation dynamical forecast models.
However, previous-generation models had shown that forecasts of winter climate anomalies in the 1960s and 1970s were less successful than forecasts of the 1980s and 1990s. Given that the more recent decades have been dominated by the North Atlantic Oscillation (NAO) in its positive phase, it is important to know whether the performance of current models would "be similarly skillful when tested over periods of a predominantly negative NAO. To this end, a new ensemble of atmospheric seasonal hind-casts covering the period 1900–2009 has been created, providing a unique tool to explore many aspects of atmospheric seasonal climate prediction. In this study we focus on two of these: multi-decadal variability in predicting the winter NAO, and the potential value of the long seasonal hind-cast datasets for the emerging science of probabilistic event attribution"[76].
"A combination of modelling studies and ground-based and aircraft measurements is used to examine the development of ice particles in convective clouds observed over the Black Forest mountains on 11 and 15 July (Huang et al.)"[77].

"Because the circulation response to external forcing appears to project strongly onto the patterns of variability, knowledge of errors in the dynamics of variability may provide constraints on the model projections. Nevertheless, because of these uncertainties, higher scientific confidence in circulation-related aspects of climate change will be difficult to obtain and for effective decision-making it is necessary to move to a more explicitly probabilistic, risk-based approach"[78]. Proximity based on weather regimes and proximity based on analogous of circulation along with the aid of conditional probability and Bayesian framework is the case which is considered in [80,81] which also is far from the teal cases in nature. Note is that the model able to do this must be GFD model like lorenz63. The model which we will use it in this study.

2.7 -Extreme snow physics and definitions

2.7.1- Physics and chemistry

Snow forms either by water vapor deposition or by the freezing of super cooled droplets. Generally, we called them aerosol. They act as cloud condensation nuclei. The speed of ice crystals formation in the super cooled clouds is deterministic for moderate to heavy precipitation in middle latitudes. the nuclei of the snow crystals in a few microns consist of

clay minerals, sodium chloride and silicate particles in addition to sea-salt. In this respect aerosols perform their indirect role.

A super-cooled droplet is in an unstable state. **homogeneous nucleation** occurs when a water droplet does not comprise external particles. Consequently, the nucleation, take place for droplets of about 1 μm , at about -41^{o} C, and for drops of about 100 μm at about -35^{o} C . Hence, only in high clouds.

For a droplet contains a freezing nucleus, the **heterogeneous nucleation** may occur.in this type of nucleation, "water molecules in the droplet collect onto the surface of the particle to form an ice-like structure that may increase in size and cause the droplet to freeze". Therefore, ice "embryo" create, from freezing nucleus scales, and with higher temperature than homogenous nucleation.

Laboratory experiment shows that "median freezing temperature increases as the size of the droplet increases". this echoes this probability that, **the larger droplet, the more likely to hold a freezing nucleus.** The freezing of cloud droplets can occur by **contact nucleation** with a fit particle. this particle is dabbed as **contact nucleus**. This process can increase the nucleation temperature also.

deposition nuclei, is a particle, upon which ice can form directly from the vapor phase. "Ice can form by deposition if the air is supersaturated with respect to ice and the temperature is low enough". If the air is supersaturated with respect to water, a suitable particle may serve either as a freezing nucleus or as a deposition nucleus. Generally, the ice formation temperature of a particle depends on the nucleation mechanism and the particle past. hexagonal Particles can be more effective nuclei. Practically insoluble particles are most effective ice nuclei. According to the literature, clay minerals is the most frequent ice nucleus in snow crystal analysis. Another is decayed plants particles. The first is inorganic with potential nucleation temperature of -15^{o}C. The second is organic with potential nucleation temperature of -4^{o}C." Ice nuclei active at - 4^{o}C have also been found in sea water rich in plankton"[14,48,50]. ***Although there is no accepted definition but according to the national weather service heavy snow equivalent to "expected accumulations of 6 inches in 12 hours (or 8 inches in 24 hours)***"[73].

2.7.2 On The General Dynamics of the Extreme Snow

A heavy, sporadic convective, snow investigated [82] in the central United States. "lower-tropospheric warm frontogenesis, and a steady stream of high (θ_e) air from the Gulf of Mexico" were the producing patterns. The narrow bands precipitation suggesting the role of warm front and a mesoscale instability in result. According to [70] Tapered channels of Intense transport of water vapor in the atmosphere dabbed as atmospheric rivers" (ARs) These ARs are mainstreams for wintertime extreme precipitation events, alike West Coast of the United States [71]. It is an accepted fact that, orographic precipitation roots in linkage of intense fluxes of moisture with the ARs, once they heading for mountainous regions [72]. According to Norris et al (2015) amplification of weather patterns extension is the kernel for extreme snow fall in western and central Himalaya. They investigate two events in January 1999 and March 2006. usual associated synoptic-scale flow patterns were amplified.

However, they also attributed "these events occurred during moderate and weak La Niña episodes, respectively" and The subtropical storm track has climatologically shifted northward over southern Asia in recent decades, so that fewer storms pass south of the Himalayas and more become terrain-locked in the western Himalayan notch". However, in a study [75] used CAM.0.4 for their experiments for planetary Rossby waves in a new "sponge methodology" applying both diabetic forcing and localized relaxation over the Pacific and outside regions respectively. Their experiments expose that the El Niño forcing is responsible for propagation of stationary waves toward east and west in the upper troposphere. "Over North Africa and Asia the aggregate undammed upper-tropospheric response is due to the superposition and interaction of these oppositely directed planetary waves that originate from the forcing region and encircle the planet". According to [76] the dominated pattern in the "Euro-Atlantic area" have been the North Atlantic Oscillation (NAO) "in its positive phase" in recent decades based on models. However, being also skillful in negative phase of NAO is subject to question. Nevertheless, based on [79] there exist a convincing connection between the NAO, with variation the North Atlantic storm track, and weather conditions of wintertime across Europe. "North Atlantic directly affect the climate and air quality of the surrounding continents"[80].

One of the causes for extreme snow precipitation is blocking phenomenon. Such as "strong North Pacific, large amplitude, long-lived blocking event that occurred during January 23-February 16, 2014"[81] the chief dynamical characteristic was "it survived an abrupt change in the planetary-scale flow when the Pacific North American pattern index changed from positive

to negative in early February. Then the blocking amplified again, and keep it up into mid-February. According to [82] blocking events are maintained by anticyclone vorticity fluxes into the blocking field by synoptic-scale eddies.

Based on [90] "locally heavy topographic snowfall", connected with "Snake River Plain Convergence Zone"(snake river plain in Idaho)-. SPCZ is a mesoscale weather system in the cold season which its occurrence depends on topographic conditions. It involves "leeward and windward flow regimes under low Fr (stable blocked flow) in a post-cold-frontal environment"[91]. Defining SPCZ needs Rossby number (Ro) and Froude number(Fr).

Convective snow-bands are another dynamical contributor for heavy snow precipitation.in the case of [91] snow-band was adjacent to a complex terrain and in conditional, symmetrical, and inertial unstable environment. As the convective circulations altered the vorticity field as well, and caused an upward inertial instability. In a complemental work [92], the dynamics and thermodynamics of two heavy snow-bands event were investigated by WRF simulations, for Calgary, Alberta, Canada. The events "not associated with **lake effects or the cold conveyor belt of synoptic-scale cyclones**" but in "different dynamic and thermodynamic mechanisms".

The first event occurs in a more mesoscale region. Actually by an aloft downstream of a jet streak in upper troposphere, conditional, dry symmetric, and inertial instability by a geostrophic frontogenesis in an unsaturated environment. The inertial instability was connected to "fast flow over upstream high terrain". The second event occurs in a region of Q-vector convergence caused by geostrophic frontogenesis –the changing rate of horizontal potential temperature gradient($\nabla_p \theta$) over time of the "-in a saturated environment , which generate "conditional, convective, and conditional symmetric instability (CSI)".

2.8- The Region

2.8.1 - Caspian Sea and its Effect

The climate system involves not just the atmosphere elements, but the five major subsystems: the atmosphere (the most unstable and rapidly changing); the ocean (very sluggish in terms of its **thermal inertia** and therefore important in **regulating atmospheric variations**); the snow and ice cover (the **cryosphere**); and the land surface with its vegetation cover (the **lithosphere and biosphere**) [14,27,50].

According to [14,50] Oceans fill 72% Earth's surface. At extreme depth they virtually reach to 11 km. By volume are equal to 2.6 km in depth of the land surface. By mass are about 250 times larger than the atmosphere. The density of sea water is linearly dependent to salinity (the concentration of dissolved salt in the water). The value of this dissolved salt is about 35 g kg^{-1} of sea water in the open oceans. Although it ranges from 34 to 36 g kg^{-1}. At the same temperature sea water is 2.4 % denser than freshwater owing to salinity.

The density (σ) of sea water deviates from 1 in g kg^{-1}. This deviation normally ranges from 1.02 to 1.03. Consequently, $\sigma = \sigma(T, s, p)$. where temperature T, salinity s, and pressure p. Although dependence to the pressure is ignorable due to very low effect. Here the salinity makes different conditions of $\partial\sigma/\partial T$ than in fresh water. In fresh water is a proportional whereas in seawater is inversely proportional. In fresh water temperature increase leads to density increase with the interval of 0 and 4 °C. In sea water, temperature increment causes density decrement. Nevertheless, near freezing point both have smaller perturbations of $\partial\sigma/\partial T$. The δs effect on density also is equivalent. In the polar oceans with larger δT and in the tropical oceans with smaller δT.

Over the most oceans, there exist smaller density in the mixed layer which stirred by wind than the water below. Most of the density gradient belongs to the ***pycnocline.*** It ranges from a few tens of meters to a few hundred meters below the ocean surface in depth. The vertical mixing cause density gradient within the pycnocline such as inversions in the atmosphere. The most important role of it, is that, the pycnocline hinders the heat and salt exchange between the mixed layer and the more abysmal layers of the ocean. At lower latitudes, it is dabbed as thermocline (i.e., the layer in which temperature increases with height), but in high latitudes particularly in polar oceans, as haloclines (layers with fresher water above and saltier water below). latitude and season effect these processes.

"Water parcels that are not in contact with the ocean surface tend to conserve temperature and salinity as they move over long distances". Hence, **we have virtually uniform water masses which their initial conditions can be trace backed to the thresholds of salinity and temperature to the mixed layers of their generating regions.**

According to the GCMs study, the most significant water masses that effect the climate in Iran and also our region of study include: The warm and saline Mediterranean outflow and *North Atlantic deep water (NADW)* Due to excessive heat energy storage during summer and sinking water respectively on those regions.

the Caspian Sea with 422,000 km^2 is the biggest salt-water lake in the world. Caspian Sea is ~79,000 km^3 by volume and its flat massive body is located between Asia and Europe. It bounds Iran, Kazakhstan, Turkmenistan, Azerbaijan and Russia stretching the shoreline to 7000 km. Its average length is 1200 km north to south and its average width is 320 km east to west [84,].

Globally the Caspian Sea embodies 14 % of the total lakes (2.7 million km^2) by area and nearly 40–44 % (79,000 km^3) of the world's lacustrine (lake) waters by volume. More than "130 rivers" supply the Caspian Sea. The north main rivers are "Volga, Ural and Terek". They give about 88 % of the total inflow. This cause a fresh inflow while the rounded 12% southern inflow has the mean salinity of ~1 % because of less inflow.

The shallow northern part reaches to 5–6 m at its extremes, while the southern part reaches to 1000 m. this create an extensive "**crypto- depression**". Although the base of Caspian is below the ocean level. Neighborhood with vast continental areas of the north with their extremely cold winters, makes the Caspian Sea to generally freezes. Although in the southern part this ice cover is unusual. Thai is the "salinity and vertical mixing of deep water" store the heat energy of summer radiation. Caspian Sea water salinity increase in winter seasons and decrease in summer seasons. This indicating the dominant role of the Volga river [84].

The deflection of the Volga River as the major contributor leads to considerable fluctuations in the lake level. The level ranges from 3 to 28 m. 3m for the last decade and 28 for the present time respectively. Although one of the main reason for this low level is the massive agricultural usage in Russia and Kazakhstan.

Climatological and meteorological conditions in the vicinity of Caspian Sea tie to the atmospheric circulation over the Eurasian continent, coastal relief and sea level fluctuation. Therefore, during winter, cold arctic air masses are dominated. although there exist air flows from mountainous region of Iran simultaneously. consequently, leads to a **high pressure convergence**. That is the reason of cyclone generation in excess of central part at this time. In addition, cyclones over the black sea and also Mediterranean Sea have their impacts in winter time. Although in summer time, anticyclone from the Siberian and also Mediterranean regions are able to move east and southwest. However synoptic process is under the influence of Azor high pressure in its maximum projection in this time.it is noticeable to say that this influence is started to wane in mid-autumn and give its sit to the northern continental air masses [84,85].

In [86] "a climate-driven ecological shifts in the Caspian Sea" depicted. They investigated those shifts using indices of NAO, ENSO, SHI (Siberian High), and the EAWR (East Atlantic-West Russia) patterns. They also used SST and "surface chlorophyll-a concentration".

In the north part we have cold continental climate and in south part we have moderate and subtropical climate. Northern and eastern winds are dominated. The north wind dabbed as "khazari". The distribution of precipitation in the region is more in the north, west and southwest while in the, northeast and east is the least. The coastal relief is the more dominant process for distribution of precipitation in the region [85,86]. Based on hydrological and hydro-dynamical study the dominant wind in autumn and winter time in Mazandaran shores are north east winds [95].

Caspian Sea level fluctuations have been connected to the climate change and deviations in the basin area. Although, a link with El Niño Southern Oscillations (ENSO) have been investigated as well. [88,89]. Based on a study by Farley and Toumi [93]"more than half of both the over-lake and Alborz mountain precipitation is a direct result of **the Caspian Sea's lake effect**". According to this study the strongest impact of lake effect on precipitation was in autumn and winter.

The existence of the Caspian Sea has a large effect on surface air temperatures due to its **thermal inertia** of the sea leads to, temperature decline in summer time and increase it in winter time. "These temperature differences impact on surface cyclonicity through their associated pressure signals".

According to [94] if hydrostatic equation defines in terms of geopotential instead of geometrical height ($gdz = d\varphi$) then variations of (z) based on pressure depends only on temperature. therefore $Z \equiv \varphi(z)/g_0$ and also the thickness of a layer $Z_T \equiv Z_2 - Z_1 = \frac{R}{g0}\int_{p2}^{p1} Td\ In\ p$ Is $\propto$ to layer mean temperature $\langle T \rangle = \int_{p2}^{p1} Td\ In\ p \left[\int_{p2}^{p1} d\ In\ p\right]^{-1}$.

Caspian Sea also impacts on flows in upper level of the atmosphere. As the summer time surface air temperatures reduction, cause a drop in the geopotential height(Z) "at 200 hPa of up to 25m", while in other seasonal time is less. (ΔZ) impacts on the zonal wind then "increase the maximum summer jet-stream speed in the region by 6.5 %". Consequently, to elude predisposition in the jet stream in GCMs we need an exact depiction of the Caspian Sea.

2.8.2- The Mazandaran province

Mazandaran located along the southern coast of the Caspian Sea and in the contiguous with the central Alborz mountains. Mazandaran province is one of the north states of Iran, actually in central-northern with approximately 24000 km^2 or mor precisely 23,833 km^2 (9,202 sq mi) lay between 35:47 and 36:35 latitudes and 50:34 longitudes. From the north it reaches Caspian Sea and from the south to Alborz mountains. The populous region with 3,073,943 people.

The region belongs a "diverse nature" along with plains, forests and snowcapped Alborz mountains. Mount Damavand has the highest peaks and volcanos in Asia. It is separated into 20 counties. It bordered with Golestan, Semnan, Tehran, Qazvin and Gilan. Mazandaran province is morphologically alienated into the coastal plains, and the mountainous areas. The Alborz mountains similar to a enormous wall enclosed the coastal strip and plains. The dominant winds are northern and eastern. Snow falls often occurs in mountainous regions, which originate several parallel rivers and streams run to the sea, unlike the rest of Iran. The rivers actually separate the lands into many isolated valleys. The regional climate is moderate and subtropical with an average temperature of 25 °C in summer time and about 8 °C in winter time. Although heavy snow fall may occur in the mountains in winter, but is rare at sea level and the downhill plains. (Figure 1) show the relief map of the region.

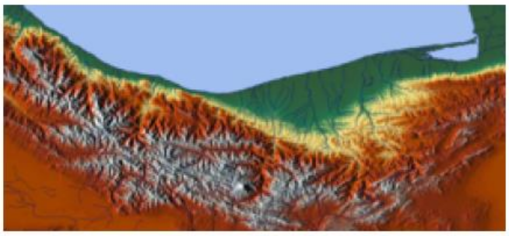


Fig1.2: The relief map of Mazandaran province

Therefor the climate is under the influence of the latitude, sea level elevation, Alborz mountains distance from the sea and also the dry and cold barren areas of Turkmenistan in south east and local air currents like sea breezes and winds along with the versatile vegetation cover.

However there also exist variety of climates, except the mild and humid climate of Caspian Sea coasts and the moderate and cold climate of mountainous regions. In western and central plains, and foothills we have moderate climate while in the altitudes of 1,500 to 3,000 m, there exists a moderate but mountainous climate, with long, cold winters, and short, mild summers. snow covers, up to last day of July [96,97,98].

This region is prone to the several disasters like the other parts of the country. But for the region of moderate climate, heavy snow falls like the year 2008,2014 and 2017 is very destructive to the infrastructures, agricultures, and livestock, the popular economical activities in the region. There exists an almost informal agreement to connect any local climate change to the large-scale phenomena and circulations. Therefore, for the purpose of our study.

2.8.3- The climate (weather regimes)

Mediterranean Sea is a focal source for precipitation in middle east as well as Iran. It also implements its extended influence on south coasts of Caspian Sea as well as Mazandaran province. According to literature, a circulation anomaly over Europe has been detected before onset of reliable numerical weather forecasting models. The so-called "omega with cold feet". This nominalization roots in omega (Ω) shape contours in 500 hPa weather maps for this region.one of main complements of this phenomenon is the northern cold air outbreaks into this region. This leads to precipitation over the western and eastern Mediterranean region. This pattern has virtually long persistence in its under-influence region. The ‘Omega-situation’ and more generally dabbed as ‘blocking’, According to Elliott and Smith, (1948). In the current literature this connects to other patterns.

Anomalies of 500 hPa height fields during wet winters in Iran, connected to the Omega pattern over Europe by Ghasemi and Khalili (2008a). Although the western branch of this anomaly is partially associated to the North Atlantic Oscillation (NAO), by Kutiel et al (2002). They also explored its eastern branch and found a NCP (North Sea-Caspian Pattern). This pattern shows a high correlation with anomalies of temperature and precipitation across the Middle East. However, its extension towards Iran is weak though it reaches its tongue to the west of Caspian Sea shores. Brunetti and Kutiel (2011) found a clear connection with the anomalies of winter time temperatures in Iran. In another study Ghasemi and Khalili (2008a) found a reliable correlation coefficient (0.43 to 0.57) between MSLP of Black Sea and the prevalent precipitation variability along the west and south west coasts.

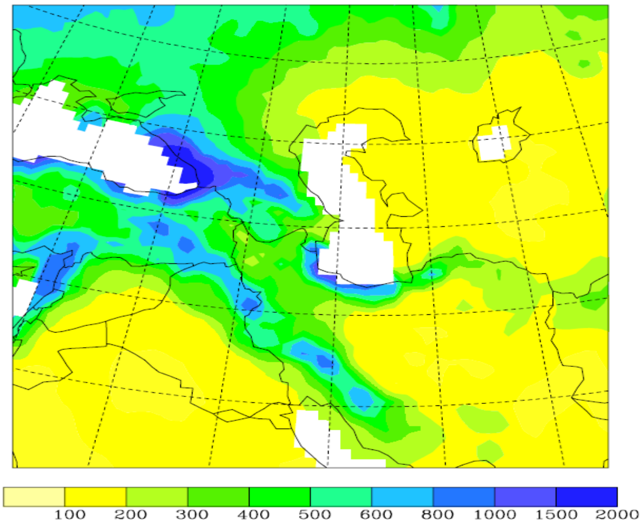


Fig. 2.2 Annual precipitation (mm) from GPCC (2013) on a 0.5^{o} grid (Fallary et al,2014).

The region climate is mainly influenced by its geographical position, geomorphological conditions- such as approaching mountains to the shore- and the predominant atmospheric circulation to the region. locating in low latitudes, leads to significant amount of solar radiation in the southern coasts.

The most outstanding geomorphological conditions to influence climate state are Alborz Mountains located in the south, the western Caucasus Mountains and deserts and steppe lands in the east and north. The main dominant air masses are Arctic cold air masses, Atlantic humid temperate air masses, continental dry and cold air masses from the Siberian anticyclone which crosses cold deserts of Kazakhstan and Mediterranean subtropical warm air masses respectively with respect to their significance (Alijani, 2014; Hejazi, 2011; Martin-Vide and Lopez-Bustins, 2006).

Monthly average temperatures from July to August differ between 24 in the north lands and 26 C^{o} in the south coats. the maximum temperature is 44 C^{o}. Although this happens on the eastern coasts with their baked lands. during winter menthes these averages are between $-10\ \circ C$ to 10 $\circ C$ in the north and south coasts respectively (Rafferty, 2011).

The total precipitation along the Iranian coasts ranges between 1850 mm in Anzali station in Gillan Province in the west and less than 200 mm in the east in Golestan Province. The Mazandaran province is in between. Whereas the average annual precipitation of the country is 250 mm/year. Therefore, this highlights the hydro-climatic and climatic significance of our study area. By precipitation Mazandaran Province is in the second level. Although The Caspian Sea basin generally lies in an area of low precipitation, the mountain ranges of the Caucasus

and Alborz are excluded. In these regions you can found high levels of precipitation. Seasonally the highest precipitation befalls in autumn, followed by winter and spring.

According to Leroy et al. (2011) highest precipitation of the area has two main reason the dominated North to North East winds and the mountain barrier along the coast. Those winds blow along with the winds above the mountains simultaneously (700 hPa and above) which blows from the west and descends down the mountains towards the Caspian. While, the updraft from the north-east surface winds does not occur exactly at the slope of the mountains but at some distance towards the sea because of the downdraft. Therefore, the more distant from the mountains the highest precipitation and the closer to the mountains the lower precipitation. Generally, the variety of wind circumstances in this region depends on its latitudinal extension, different morphological characteristics around the sea and different converging weather systems to this area. The dominant westerly sat higher levels (700 hPa or higher) descend down the mountains and meet the north-easterlies at the surface over the CS. This is a reason for cyclone generating of the Caspian Sea.

2.9 Summery

The importance of extreme events is an undeniable fact in the new perspective of the climate science. This issue is so much clear from the bulk of research papers and books in the field. But the after almost 40 years of research and simulation with the supercomputers and growing the software as well as hardware developments the results is nor really satisfying. Although based on the extreme whether attribution report from NAS of the US the main problem in issue is the lack of the related data. It is proposed in this study that in addition to the problem with the data the main problem is in our understanding and then our physical and then mathematical presuppositions which by itself is rooted in the wrong epistemology of the field.

Therefore, in this chapter it is tried to at first to stablished a sound theoretical foundation for the problem at hand. This foundation is gained by the comprehensive consideration and investigation of the far and near concepts to the problem. Hewer because of the lack of the similar researched and also interested researchers in this context in Iran this made by a lot of efforts. This is because it is clear that the problem at hand needs a bunch of related experts to perform.

But by the aid of craziness to the physics the project started by some reports from IPCC and similar works. Then go on with the related and introductory books. Then to the related papers

and video lectures on the web. And ultimately to the result that there is critical need to turn the page and look differently.

This different look can be gained by the aid of climate modeling based on the Hamiltonian, chaotic and stochastic reduced and truncated dynamical models. Reduced and truncated as we are considering a problem in a large temporal and special scale. And this because of decreasing the complexity as there is no universal model with characteristics which is needed to the problem at hand in this study if you wanted to look differently.

This way of working is mainly associated to the scrutiny of the physical basis of the climate models and as well as climate system by itself. Based on that consideration it is reveal that;

How can we expect to reach to a point of success or even near to it with conservation, equilibrium and determinism? While the system at hand I dissipative, non-equilibrium, stochastic, chaotic and probabilistic! Chapter 3 is dedicated to the methodology and more details of the issue as is needed to a sound framework of study as we know different frameworks even in a same subject tends to different results then misleading!

The Caspian Sea is a main climate driver in the region simply because it is fixed. Thereby directly control the weather regime of the province and surrounding lands. This makes the region different from Mediterranean climate regimes although the regime would affect the region in its exceeding. The region climate is mainly influenced by its geographical position, geomorphological conditions- such as approaching mountains to the shore- and the predominant atmospheric circulation to the region.

The shallow northern part reaches to 5–6 m at its extremes, while the southern part reaches to 1000 m. this create an extensive "**crypto- depression**". The deflection of the Volga River as the major contributor leads to considerable fluctuations in the lake level. Climatological and meteorological conditions in the vicinity of Caspian Sea tie to the atmospheric circulation over the Eurasian continent, coastal relief and sea level fluctuation. Therefore, during winter, cold arctic air masses are dominated. The dominant winds are northern and eastern. Snow falls often occurs in mountainous regions, which originate several parallel rivers and streams run to the sea, unlike the rest of

CHAPTER III

Method and Data

3.1- Motivations and Preliminaries

Mandelbrot (1982) declared that, nature has a "irregular and fragmented patterns" which cannot be display by the Euclidian geometry. Nature exhibits a higher and different level of complexity. Then it is a need to have a "family of shapes" i.e. fractals. Most helpful fractals contain probability then have statistical (stochastics) regularity and irregularity. So Identical fragmentations (irregularities) in their scales [139,140]. Many of natural dynamical phenomena is very far from reaching to equilibrium point as they are "open and extended" systems with interacting with their environments. Then the traditional Hamiltonian dynamics is not able to governs the dynamics of them. That is stationary scale of invariant probability distribution (such as PDF and FPE) is able to describe the correlations (causations) of these systems [139,140].

There is a continuous argument for predictions of climate change owing to human induced perturbations in greenhouse gases and concentrations of aerosol (IPCC, 2013). Among different variables, variabilities of time series and spatial distribution of temperature and precipitation are significantly important in climate studies. Therefore, there is an impervious need for unequivocal long time series of data. Instrumental and site altering are the very potential causes for any misleading changes. Then they must be detected and removed or cured [4,5,6].

Extreme snow precipitation for a region with moderate climate is a rare event. Therefore, it belongs to the realm of probability not frequency. According to [24] the first step is to found a "mathematical framework (naturally a probabilistic framework) where the meaning of rare event becomes precise". Then a suitable notion. Therefore, it is natural setup a probability space χ fitted out with a σ-algebra, β of events and a probability measure P designed to quantify the probability of the occurrence of any event $A \in B$.

Bernoulli distribution says a discrete probability distribution whose PMF is equal to P (X = 0) = 1 p and P (X= 1) =p has a binomial distribution B(1,p).here 0 as a fail and 1 as a success. Although according to [1] we can call it y=1 and y=0. here Y is defined result of an observed extreme situation in excess of a **threshold (u)** of a relevant **climate index (Z)**, both in a large extent. According to Stone and Allen (2005), evaluating these extents to base on external

forcing, mathematically f ∈ F has p(Y). This is the initiative point for factual (with external forcing so natural) and counterfactual (without external forcing so not natural) in extreme event attribution study. Therefore, if $p_1 = P(Y = 1|F = 1)$ indicates factual conditions, even though $p_0 = P(Y = 1|F = 0)$ specifies counterfactual conditions. This by itself defines the FAR (Fraction of Attribution Risk), $FAR = 1 - \frac{P_0}{P_1}$.

The fraction specifies the amount of likelihood for an event that is attributable to the external forcing (F). Stott et al. (2004) interpreted the Causal factors of European summer temperature anomalies based on FAR and its uncertainty, associated with model and sampling errors, as there exist a very high likelihood that anomalies more than a threshold of 1.6°C is attributable to human induced external forcing [5]. this conventional frame work stem in the standard theory of causality. Although there exist several definitions of causation depending on counterfactual logic, pearl (2000). It is still based on conditional probability of 18th century but just extended within the probabilistic and graph framework of casual counterfactual theory. Actually this pave the way for the application of counterfactual methodology to actual datasets and then **reliable causal inference** [39]**.** According to [5] there exist a conspicuous shortage in this context in the current literature.

To have causal hypothesis basically we need tow set of data, observational and experimental. the difficulty starts with the application; then assumed a series of data, experimental or observational, so now what is the causal conclusions? And what is the level of associated confidence? [40].

Over all, the paradigm of understanding emergent properties of the complex system via the bottom-up agglomeration and interaction of small scale processes has become dominant in climate science. While it is conceivable that a top-down principle –this can be equal to the of Cause' - could be found that provides better predictions and thus supplants climate modelling, no such principle has yet been discovered and we think it unlikely that one will emerge. Meanwhile, climate models continue to increase in skill and scope(Knuttietal.2013) and the challenge to intelligently hall of defaces that resource in the most effective and rigorous way possible to better understand the world continues to grow [99].

3.2- Two near worlds instead of one far

The **factual** world is close to the one in which we live, either in terms of anthropogenic/natural climate forcing or in terms of temporal proximity (e.g. the last decades). The **counterfactual** world contains only natural climate forcing, or temporal remoteness (e.g. beginning of 20th century (1900–1950) vs. recent decades (1950–2016)). It is common to define an extreme event (in either worlds and universes) when a reference threshold has been reached or exceeded (e.g. $y >= U$). In other words, we are dealing with a "class of events" includes the ensemble of weather types for which the threshold can be reached". In this work, **we assume that such an extreme event is reached during a spell of atmospheric circulation because of strong fluctuation in the absorption and emission of the thermal and solar radiations**

Based on probabilistic framework the goal of extreme event attribution is to determine how the probability of an extreme event differs between **factual** and **counterfactual** words. But the problem will rise when you encounter with a rare event. In fact, achieving this goal is trivial if a rare event occurs in one of the worlds and is impossible in the other. In practice, this does not happen for most extreme events that have occurred in the past decades, because there are often historical examples of such events (e.g. most European winter storms, European heatwaves). Thus, we assume that a given extreme or rare climate event has a probability of occurrence p_1 in **factual**, and p_0 in **counterfactual** worlds.

3.3- Essential Physics of the method

According to (L. D. Landau and E. M. Lifshitz, 1966**)** Energy dissipation occurs in fluids on motion due to thermodynamic irreversibility. Because of internal friction(viscosity) and thermal conduction. Viscosity coefficients are $\nu = \eta/\rho$ m where ν is the kinematic viscosity and η is the dynamic viscosity. These are the viscosity coefficients in naiver -Stock equation as well. For air η (g/m sec)=0.00078 however ν(cm2/sec)=0.150. as air is an incompressible fluid viscosity causes dissipation of energy which consequently transformed to heat therefore conservation of energy cannot be hold in a viscous fluid such as air due to "irreversible process of energy dissipation". Also thermal conduction is another way of energy transfer in this sort of fluids. κ corresponds to the thermal conduction [114].

According to [114] naiver- stock equation can be as follows in a concise way**;**

$\frac{\partial v}{\partial t}$ +(v.∇)$v = \frac{1}{\rho}\nabla\, pt\ \frac{\eta}{\rho}\ \Delta v.$ (1.3)

However according to Palmer (2016) it can be write as

$$\rho\left[\frac{\partial}{\partial t} t\ u \cdot\ \nabla\right].u = \rho g - \nabla p +\ \mu\nabla^2 u \quad (2.3)$$

For more see [94] chapter 2. The thermal conduction equation or Fourier equation is as follows as well; $\frac{\partial T}{\partial t} = \chi\Delta T$ where χ is the thermal conductivity ans defined as; $\chi = \frac{k}{\rho c_p}$

Since the air is a mixture of fluids it is not homogenous. Concentration of a matter describes its composition by $c = \frac{m_{ri}}{M_{ri}}$

The distribution of concentration will alter through the fluid in two different ways; Macroscopic and Microscopic motions. In Macroscopic motions the composition will remain unchanged in every small partition as the move in sub-portions the move in the whole body. Consequently, we have a pure mechanical mixing. This leads to variation of concentration with time.

If we ignore χ and ν ,the variation of concentration will be thermodynamically a reversible process and does not leads to the dissipation of energy.

In Microscopic motions the variation of composition happens by molecular transfer of the components from one part of the fluid to the other part. "the equalization of the concentration by this direct change of composition of every small portion of the fluid is called diffusion". In other words, homogenous concentration by microscopic motion. diffusion is an irreversible process and is one of the sources of energy dissipation in a mixture of fluids as well as χ and ν as mentioned before. So totally the energy dissipation sources in a mixture of fluids consisted of **viscosity, thermal conduction and diffusion.**

Velocity in a mixture of fluids will define as $\frac{\partial \rho}{\partial t} + div(\rho v) = 0$. In fact, this the continuity equation which "express the conservation of mass for a fluid" [94,114]. Then we can define the velocity as "the total momentum of unit mass of fluid".

According to the above notes we are dealing with *a continuous and mostly in compressible and mixed fluid so the induced charges(momentum) are irreversible and then deterministically unpredictable.*

On the basis of [115] every object holds a large amount of electric charges which is the natural characteristics of its fundamentals particles so it is a programmed property for those particles. This charges can move through the interested matter by the property of conduction. That is due

to the structure and electrical nature of atoms: i.e. protons (+), electrons (-) and neutrons (not+ and not-) and also the fact that protons and neutrons are bounded tightly in a central nucleus. In fact, **the charge transfer is the causes of heating.**

Another fact is that angular momentum is a fundamental quantity because it is an inherent property of atoms, molecules and their constituents. These properties due to rotational, vibrational and spinning frequency causes momentum transfer from microscopic scale to macroscopic scale so then vorticity to velocity in a mixture fluid like Atmosphere.

We have used a new attribution method in a systematic way. This approach is based on causality theory [108]. This method is efficiently using for weather and climate event in 'near real time'. Data assimilation has a broad usage in several field of science. Here we used it for our causal attribution method. Therefore, generally we can call our approach detection and attribution by data assimilation methods.

Although specially we are detecting and attributing an extreme weather event (e.g. extreme snow fall of Mazandaran province) but this aim is gained by the ensemble kalman filter as one of the kalman filters.

Unfortunately, there is no accepted and general model for the purpose of our study. The reason is that our event is a rare event in the region of mid climate regime (e.g. extreme snow for the Mazandaran province).it could be say that the main reason for this surprise is the presence of Caspian Sea with its Mediterranean like water cycle in addition with the 1000 deep of the sea in its south and south west. Salinity rise in the summer time leads to gaining more heat and heat capacity rise. This holds the summer heat in the southern deep parts and then directly or indirectly affects the climate of the southern regions of the Caspian Sea. Although this seems very clear but it needs more researches!

Attribution can be generally defined as; finding casual connection between climate forcing & observed responses. This is called detection and attribution. The main problem in hand is "generating causal information" for extremes because of practicalities for public, legal & scientific context [104,105,106,107]. In practice this is a probability approach so it evaluates the probability of event occurrence by considering the intensity of extreme climate forcing.in literature the usual climate forcing to consider are GHG emissions, ozone or aerosol condensations an also solar radiation and so on so forth.

In this work we just decided to consider solar radiation. This is because, we used Lorenz 63 model which basically is a 3-D empirical model. However, its dimensions can be expanded to 9-12-D [109] which is our future perspective but for this job. Actually the Lorenz 63 model is truncation of Naiver- Stock equations for fluid motions in non-linear dynamical systems.it

treats as boussinesq approximation and approximates a 2-D dissipative Rayleigh-benrad convection [109].

3.4- The process of the method and extreme snow fall of Mazandaran province

For our aim (attribution of extreme snow falls of Mazandaran province in near-real time view) we need two worlds to compare probability of occurrence. We call them factual and counterfactual as usual in the context [110]. the former with external forcing and the latter without. This process performed "in an *ensemble of model simulations* representing the observed climate condition" which simulate probability of occurrence of the event in the factual world which is the real world with the probability of occurrence of the same event in the parallel *ensemble of model simulations* which is the none real alternative world. Ensemble in word means a bunch or a group. Here means a set of simulated climate conditions. Its roots turn back to the statistical physics and specially to the classical statistical mechanics and the theory of Canonical ensembles and also its usage for the Gibbs ensembles [111].

By these bases we have p1 as the probability of the event occurrence in factual world and p0 as the probability of the event occurrence in the counter factual world. actually this is not a new framework in event attribution but it has been long from [112] to [108 &1] to develop for instance of course! By the aid of [112] which is the historical work of Stone and Allen we can define FAR which is a fraction of Attribution risk.   FAR=1-p0/p1.in fact FAR "has interpreted as the *fraction of likelihood* of an event which is attributable to the corresponding external forcing in casual attribution framework in the past decades [105].

Based on [113] which also a remarkable work in casual theory in terms of the real assumptions the FAR can be interpreted as the probability of necessary condition(causation), we can call it NP (necessary probability). This is a casual link between the event occurrence and its corresponding external forcing. Although according to pearl [113] this is one of the two requisite conditions (causations). Consequently, we sufficient condition as well. So SP (sufficient probability). then we can say: NP=1-p0/p1 and SP=1-1-p1/1-p0. According to them NSP=p1-p0 as necessary and suffusion probability conditions. These are come from pearl (2000) which originally formulated there. Therefore, in causal Attribution framework finally we need to calculate p0 and P1 to conclude our likelihood as a final remark.

According to [24] most climate simulations by models used large ensembles in order to approximate p1 and p0. they used different kind of methods but the common method is the EVT which is an abbreviation for conventional Extreme Value Theory. In fact, this is a *general statistical approach*. Based on [24,108,112] the problem with this approach is that it has high

cost of computation and more significant is that its interpretation is not "acceptable, usable, manageable & systematic ".

3.5- The Deterministic Lorenz-63 model

This is a 3-D dynamical system. Originally Lorenz equations were derived by Saltzman (1962) as a truncated model of thermal convection in a box

$$\frac{dx}{dt} = \sigma(y - x),$$
$$\frac{dy}{dt} = x(\rho - z) - y,$$
$$\frac{dz}{dt} = xy - \beta z. \quad (3.3)$$

Where σ (Prandtl number): $\frac{\upsilon}{\alpha}$, which is a ratio of momentum diffusivity over thermal diffusivity. Where υ is the kinetic viscosity or the same momentum diffusivity which is equal to the nominator of ($\frac{\mu/\rho}{k/(C_P\rho)}$) that is another form of defining of σ this dimensionless number also can be written as $\frac{C_p\mu}{K}$. The parameter of is the same of the section (3.3).

ρ (Rayleigh number): also is a dimensionless number in the fluid dynamics and is defined for a fluid which is driven by the buoyancy and consequently yield a free convection motion in a fluid. The system of motions of the fluid is identified by it. The heat transfer also can be derived by this ratio then in a certain lower values this transfer is by the conduction rather than convection. For more information, [94,138]. And β is a geometric factor which is defined by the Lorenz (1963) which is a positive number (β > 0) and identified by $\beta = \frac{4}{1+a^2}$. *The role of these three numbers is that the parameters which is defined for the system to consider is proportional to them* i.e.(X,Y,Z $\propto$ $\forall$ ($\sigma.\rho.\beta$)).

These make us the Lorenz attractor which is rooted in a truncated system of equations for a 2-D flow of fluid with "uniform depth" (H), with an imposed temperature difference ΔT, under gravity (g), with buoyancy (α), thermal diffusivity K , and kinematic viscosity ν.

They are commonly supposed to be positive. The left side can also be represented as ($\dot{x}.\dot{y}.\dot{z}$) which explicitly shows that the system is autonomous. In this context x is proportional to convective intensity, y to the difference in temperature between ascending and descending currents, and z to the vertical temperature profile’s deviation from linearity. However, theses representations are general and based on the context of study theses representations can differ.

These equations also arise in studies of convection and instability in planetary atmospheres. Also Malkus' water wheel indicated that this strongly truncated deterministic system from Naiver-Stock equations for fluid which by itself roots in newton second law for the fluids, turns out to have "interesting" erratic and unpredictable dynamics [116,117,118].

3.5.1- Properties of these equations (brief overview)

Since this study is not about these properties or their usage in the context of study but it just uses the foundations to build a conditional probabilistic framework it did not enter to the details of the topic and put it away for future studies. However, it is tried to propose the important properties as an outline below;

These three ODEs are nonlinear, symmetric (invariant under $(x,y) \rightarrow (-x,-y)$) and possessing *contracting volume*. The last is due to the fact that this system of equations is representing a *dissipative* dynamical system which its volumes in phase-space contract under the flow. Although another property is that of its fixed points - $(x*, y*, z*) = (0, 0, 0)$. It is a fixed point for all values of the parameters. For $\rho > 1$ there is also a pair of fixed points $p^{\pm}$ at $x* = y* = \pm\sqrt{\beta(\rho - 1)}$ and $z* = \rho-1$. These conjoin with the origin as $\rho \rightarrow 1^{+}$ in a ***pitchfork bifurcation.*** Although according to Wikipedia article of Lorenz system; "a pitchfork bifurcation occurs at $\rho=1$, and for $\rho>1$ two additional critical points which correspond to steady convection appear at:

$$\left(\sqrt{\beta(\rho-1)}, \sqrt{\beta(\rho-1)}, \rho-1\right) \text{ and } \left(-\sqrt{\beta(\rho-1)}, -\sqrt{\beta(\rho-1)}, \rho-1\right) \tag{4.3}$$

The stability occurs only when;

$$\rho < \sigma\frac{\sigma+\beta+3}{\sigma-\beta-1}, \tag{5.3}$$

However, this is hold only for positive ρ if $\sigma > \beta+1$. The important note is that at the critical value, both of these equilibrium points lose stability through a ***Hopf bifurcation***. Actually *bifurcation gives a summary of the essential dynamics to yield a global view of over a range of parameter values.* Then *allow compression between periodic and chaotic behavior at the same time*. Therefore, the more practical explanation of the matter would be that, *in dynamics a change in the number of solutions to a differential equation, as parameters varied is called* ***bifurcation*** [116,117,118].

For Lorenz-63 the mentioned constants are as; $\rho=28$, $\rho=10$, $\beta=8/3$. These values are defining by Edward Lorenz (1963). the Lorenz system of ODEs has chaotic solutions (but not all

solutions are chaotic). Even more elementary model systems can give a chaotic behavior. As stated above these are stated in the form of difference equation rather than differential equation. Almost all initial points will tend to an invariant set – the *Lorenz attractor* – a *strange attractor* and a *fractal*. Its Hausdorff dimension is estimated to be 2.06 ± 0.01, and the correlation dimension is estimated to be 2.05 ± 0.01[15,117]. Although the exact Lyapunov dimension formula of the global attractor can be found analytically under classical restrictions on the parameters as;

$$3-\frac{2(\sigma+\beta+1)}{\sigma+1+\sqrt{(\sigma-1)^2+4\sigma\rho}}. \tag{6.3}$$

Although the action of the differential equation on the attractor is described by a fairly simple geometric model analysis of the Lorenz attractor is not straightforward.

The uniqueness theorem in the context of the ODEs of the Lorenz system means that trajectories cannot cross or merge, hence the two surfaces of the strange attractor can only appear to merge. Therefore, Lorenz (1963) concluded that "***there is an infinite complex of surfaces***" where they appear to merge. Today this "infinite complex of surfaces" would be called a ***Fractal***. A fractal can be defined as a set of points with zero volume but infinite surface area. This is the geometry of the model and its strange attractor

3.5.2- Exponential divergence of nearby trajectories

The motion on the attractor displays sensitivity to initial conditions. Two trajectories starting very close together. After a while they rapidly diverge from each other, and would have different paths. The note is that this makes the long-term prediction impossible which also an absolute fact in the climate system. Another point is that small uncertainties are amplified enormously fast which is also is interstice in climate system because of its chaotic and random behavior.

Sensitivity to initial conditions on Lorenz attractor in numerical studies of the Lorenz attractor, one finds that $\|\delta(t)\| \sim \|\delta_0\|\, e^{\lambda t}$, where $\lambda \sim 0.9$. Hence neighboring trajectories separate exponentially fast. The number λ is called the ***Lyapunov exponent***. In fact, it is a mathematical approach for computing the rate of separation and its positive value shows the unpredictability. However, there are actually *n* different Lyapunov exponents for an *n*-dimensional system [116.118,120].

The first is that when we Consider the evolution of an infinitesimal sphere (in phase space) of perturbed initial conditions. During its evolution the sphere becomes distorted into an

infinitesimal ellipsoid. Let $\delta_k(t), k = 1,2,3...n$ denote the length of the kth principal axis of the ellipsoid. Then

$$\delta_k(t) \sim \delta_k(0)e^{\lambda_k t}, \tag{7.3}$$

where the λ_k are the Lyapunov exponents. For large t, the diameter of the ellipsoid is controlled by the most positive λ_k. Thus our λ above is actually the largest Lyapunov exponent. The second is that λ depends (slightly) on which trajectory we study. We should really average over many different points on the same trajectory to get the true value of λ. When a system has a positive Lyapunov exponent, there is a time horizon beyond which prediction will break down [116.118,117].

3.5.3- Chaos in Lorenz-63 model

Although, there is no universal definition of the term "chaos" -but the most common definition is: *Chaos is non-periodic long-term behavior in a deterministic system that displays sensitive dependence on initial conditions [116].*

Non-periodic long-term behavior means that there are trajectories which do not settle down to fixed points, periodic or quasiperiodic orbits as $t \to \infty$.

Deterministic means that the system has no random or noisy inputs or parameters Its irregular behavior arises merely from nonlinearity of the system.

Sensitive dependence on initial conditions means that nearby trajectories diverge exponentially fast, i.e. the system has at least one positive Lyapunov exponent [119].

For more clarity lets define the common terms as below;

Trajectories are resisted toward infinity, and never return then no equilibrium. Hence a fixed point of the system is the infinity.

An attractor is a set of points to which all neighboring trajectories converge such as Stable fixed points and stable limit cycles.

Strange attractor is an attractor that exhibits sensitive dependence on initial conditions.

When the long-term behavior is periodic the dynamics are not chaotic. However, here the dynamics do exhibit sensitive dependence on initial conditions but is still unpredictable. That is transient chaos shows a deterministic system can be unpredictable, even if its final states are very simple. This is familiar from our everyday experience and in weather and climate phenomena [109,117,118,119].

The above brief overview of the chaotic dynamical system and the celebrated Lorenz-63 are the basis of our interpretation for the counterfactual world in our probabilistic and chaotic view of the problem proposed here

3.6- The mathematical (logical) foundations of the method

"Probabilistic causality is a branch of philosophy that attempts to explicate causal relationships in terms of probabilistic relationships. This attempt is motivated by several ideas and expectations. First and foremost, probabilistic causality promises a solution to the centuries-old puzzle of causal discovery - that is, how humans discover genuine causal relationships from bare empirical observations, free of any causal preconceptions. Given the *Humean* dictum that all knowledge originates with human experience and the (less compelling but fashionable) assumption that human experience is encoded in the form of a *probability function*, it is natural to expect that causal knowledge be reducible to *a set of relationships in some probability distribution* that *is defined over the variables of interest*" pearl (2000).

Standard probabilistic accounts of causality assume that, in addition to a probability function P, we are also given the temporal order of the variables in the analysis therefore we Supposed y_t represent the d-dimensional vector for our observations with discrete times progression {t = 0, 1..., T}. Then, y is equal to a set of $\{y_t : 0 \leq t \leq T\}$ which corresponds, to a set of our all observations over a time interval from 1987 to 2017 covering the Mazandaran extreme snow, min temperature and net radiation of DJF over these 30 years of NOAA satellites observations. The selection of these data was for the reason of my general view to the considering problem. However, the more important motive was the future plan of research. Also according to the literature none of them can exactly what is the main cause of the extreme event and almost all of them achieved the *forcing of all*.

In the contemporary probabilistic framework for D&A, the observed trajectory y is a realization of a random variable represented by Y which is equal to the set of $\{Y_t : 0 \leq t \leq T\}$, i.e. ∃ an $\omega \in \Omega$ such that $Y(\omega) = y$. here Ω represents the sample space ∀ possible outcomes and Also includes errors related to the observations, as well as internal variability.

This is a somewhat subjective choice in picking up a subset $F \subset \Omega$ for defining the event occurrence in event studies considering attribution of extreme weather and climate events with respect to both necessary and sufficient causal conditions. However, this selection directly can shake our causal conclusions. The loose definition of the event, the low threshold of evidence and the tighter the severe threshold.

Several factors should be necessary to consider for occurrence of a specific event such as our case of study. For such class of rare events in detection and attribution of extreme events, either

p0 and p1 remain very small. Our final aim was confirming our results by the necessary and if it possible the sufficient conditions for our definition of the event at hand as follows;

$p_i = P(\varphi(Y) \geq u)$

is the usual definition for event occurrence in detection and attribution context. Where φ(Y) is a scalar index, exceeding a threshold u. because of our usage of sub (i) then, i = 0 represents counterfactual world and i = 1 represents factual world. I used PDF (probability density function) for the definition of the event of interest, i.e. The PDF is our scale for the detection then attribution with the value of 0 to 1. A suitably restricted definition of the occurrence of the interested can be a set as follows;

$\{\omega \in \Omega \mid Y(\omega) = y\}$

Which stands for their related PDFs. This (PDF) i.e. f(Y(ω)) of Y can be generally assumed, to be continuous which means it contains no singular δ-function.

"Assessing the likelihood that one event was the cause of another guides much of what we understand about (and how we act in) the world. But causation has two faces, necessary and sufficient. It says, the probability that event x was a necessary or sufficient cause (or both) of another event y. The standard counterfactual definition of causation (i.e., that E would not have occurred were it not for C) captures the notion of "necessary cause."

Although the distinction between necessary and sufficient causes goes back to J. S. Mill (1843), it has received semiformal explications only in the 1960s -via conditional probabilities (Good 1961) and logical implications (Mackie 1965)".

so we have a "parallel notion of causation", for the "production of the effect" *necessary and sufficient* [1,5,108,113].

"Even in the absence of confounding, probabilities of certain counterfactual relationships cannot be identified from frequency information unless we specify the functional relationships that connect causes and effects". The following definitions which is necessary for clarification of the method is derived from [113,108]

3.6.1- Probability of Necessity

Let X and Y be two binary variables in a causal model M. Let x and y stand (respectively) for the propositions X = true and Y = true, and let x' and y' denote their complements. The probability of necessity is defined as the expression

$$PN = P(Y_{\acute{x}} = F \mid X = T \,.\, Y = T \,; = P\left(\acute{Y}_{\acute{x}} \mid x.y\right)$$

Where T is for true and F is for false. In fact, PN holds for the probability of y_i' (that event y would not have occurred in the absence of event x), in condition that x and y actually did occur.

3.6.2- Probability of Sufficiency

$$PS = P(Y_X \mid \acute{y} \,.\, \acute{x})$$

Here PS measures the "*capacity of x to produce y"* and, since "production" infers a transition from the absence to the presence of x and y, P (Y_x) is existed given that either x and y be absent. Thus the probability that setting x would produce y in a situation where x and y are in fact absent.

3.6.3- Probability of Necessity and Sufficiency

$$PNS = P(y_x \,.\, \acute{y}_{\acute{x}})$$

PNS holds for the probability that y would return to x mutually i.e. *measures both the sufficiency and necessity of x to produce y*.

Although one can define more notions associated with these elementary notions there are other counterfactual quantities. But all of them Also can be inferred by the quantities are mentioned [113,108].

3.6.4- PDF and probability of occurrence

Based on the above theoretical considerations, the PDF of Y at a single point Y = y is evaluated instead of evaluating the probability P(φ(Y) ≥ u) as the problem of consideration wants this and it is easier as well.

According to (Gardiner, 2004) if Y be for an autoregressive process we can have; $Y_{t+1} = A\, Y_{t+wt}$

where w_t is an i.i.d noise with a known PDF. Here stationarity is guaranteed by A. by this and above presumptions and logics we have;

$$f(y) = \prod_{t=1}^{T} g(y_t - Ay_{t-1})\pi(y_0) \qquad (8.3)$$

Where π(.) is the prior PDF as well. This PDF is defined on the initial state Y_0.

Notice that if we consider P(φ(Y) ≥ u) then f(y) should be computed needed to integrate and also the expensive Monte Carlo method of simulation.as mentioned before this the common method EVT.

As we said before we have two worlds to consider Factual and Counterfactual. Also we defined these two worlds by the Lorenz 3-D model of 63 then said that these two modeled worlds must

be computed numerically because of our long time of ensemble of simulations. Now let's turns back to that point and continue with the more fundamental and practical view.

The *Random noises* we added to our three ordinary differential equations were Independent, and Identically Distributed (i.i.d) with zero mean which is dabbed as *white noise*. White noise is a time series with the same variance of its variables and also with the zero correlation with other values in the series.

This roots in the randomness of process of the study which is mentioned in the first part of this chapter and chapter 2 as well. Here we say that actually this is the well-known ***Brownian Motion*** which almost all of this sort of process and framework is come from which also known as winner process. The former in the microscale of the matter and the latter in the probability theory.

The *Fokker –Planck equation* gives time evolution of PDF of a system. There exists a connection between the FPE and SDE which is now made by adding random white noises to the former ODE. This is because f(y(t)) has a conditional probability density, p (y, t | y_0, t_0). Then one can calculate the FPE corresponding to the related SDE or vice versa. Actually here the aim is the use of FPE instead if SDE indirectly. However, to obtain a full view of diffusion process both may be required. IT was Fokker which investigated the Brownian motion in a radiation field. The SDE system defined above is stationary, then the PDF of the state vector is independent of x_t and x_0 - x by itself represents the required vectors of the model i.e. X, Y, Z, this can be said as x_t as a vector of random process which has an appropriately defined PDF- afterward a sufficient long time (t). This PDF commonly calculated as the numerical solution of associated the FPE. In fact, it is the mean over Ω of the sample measures obtained for each realization ω of the corresponding noises of the related SDE.

As the essence of the framework is on the conditional probability the stochastic process explained by it which satisfy the corresponding FPE is actually identically equal to SDEs of the Ito calculous which by itself (which can represent a mathematical summary of the whole process as well) is;

"let $f(t)$ be a step function in $L_w^2\ [a.B]$, say $f(t) = fi$, if $t_i\ \le t\ < t_{i+1}. 0 \le i \le r-1$ where $a = t_0\ \le\ t_1 < \cdots < t_r = B$. The random variable $\sum_{k=0}^{r-1} f(t_k)[w(t_{k+1}) - w(t_k)]$ is denoted by $\int_{\mathbf{a}}^{\mathbf{B}} \mathbf{f}(\mathbf{t})\mathbf{dw}(\mathbf{t})$"

where K_n is a continuous function. This process named as stochastic integral of f with respect to the Brownian motion W (winner process) it is Also called the Ito integral. The solutions are continuous non-differentiable everywhere with the probability of 1.0.

The important note to remember here is that as the probability we wish to calculate originally is in the form of transition probability function, that is;

$p(s.x.t.A) = \int_{R^v} p(s.x.\lambda.dy)p(\lambda.y.t.A)$ for any $s < \lambda < t$

Which is the well know Markov transition function which also satisfy the chapman-Kolmogorov equating that is the one of the basis of Kalman Filters.

By these we can have 1-D FPE as follows;

$$\frac{\partial f(x,t)}{\partial t} = -\frac{\partial}{\partial x}[A(x,t)f(x,t)] + \frac{1}{2}\frac{\partial^2}{\partial x^2}[B(x,t)f(x,t)] . \tag{9.3}$$

However if it is write as $\frac{\partial p}{\partial t} = -\nabla \cdot (f(x.t)p) + \sum_{i.j} \frac{\partial^2}{\partial_{x_i}\ \partial_{x_j}}\ (Q/2)_{ij}\, p = L_{FP}(p)$ (10.3)

Which then would be an advection- diffusion equation. This also shows the difficulty of numerical integration as in Naiver-Stokes equations. Where $Q = g(x.t)g^T(x.t)$

where x corresponds to the required vectors as mentioned above [119,120,121,122,125]. This also important to note that as this study is basically in conditional A&D approach its FPE must be conditional as well. This will be considered in the next section.

3.7- The two models' system of equations and their solutions

(For this study; nonlinear data assimilation in nonlinear dynamical system).

The 3-D Lorenz -63 dynamical model is a system of ordinary differential equations which can also write as follows;

$dx = \sigma(y - x)dt$. $dy = (\rho x - y - xz)dt$. $dz = (xy - \beta z)dt$ (11.3)

The problem will rise in finding a common solution for this system of three ODEs as because uncertainty in initial conditions and the long time interval the analytical solutions do not able to satisfy our aim. As common in this sort of occasions the only alternative is the usage of numerical methods.

Actually a nonlinear dynamical system is of the form, x = f{x) where $f : R^n \rightarrow R^n$ is sufficiently smooth. The most significance difference between nonlinear and linear systems is that, generally there exist no "closed formula" for an exact or analytical solution as in linear case. Then in most cases numerical techniques, is the only alternative to approximate solution

[123,125]. However, it is important to note that the FPE corresponding to this unperturbed model is as follows

$$\partial_t p(x.t) = -\nabla \cdot (p(x.t)F(x)) \tag{12.3}$$

This known as the transport equation and also known as the Liouville equation which is a counterpart of the FPE for SLM model. "This provides the probability density at time t of S(t)x when the initial state x is sampled from a probability measure that is absolutely continuous with respect to Lebesgue measure; here $\{S(t)\}t \in R$ is the flow of $\dot{x}$= F(x), for some sufficiently smooth vector field F on R^d ". This *smoothing aspect of random perturbations* is often useful in the theoretical understanding of any stochastic system. Actually this shows the "corresponding non-degeneracy conditions" which regularize the stationary solutions of the counterpart of the *Fokker–Planck equation in the absence of noise* [125,126]

In the case of LM-63 the most common and exact approximation is yielded by the well-known Runge-Kutta Method of 4th order. Although the problem here is the initial value problem i.e $y' = f(x.y)$. $y(x_0) = y_0$ and the aim is to give approximate values y_i for the unknown function $y(x)$ at a chosen set of interpolation points x_i i.e interpolating by nodes with the equal distance along with the previous given step size h. this can be shown as $x_i = x_0 + ih \ and \ (i = 0.1.2.3 \ldots\ldots)$. According to this we have Runge-kutta method as bellow;

Here the equation $y'(x) = f(x.y)$ determines at every point $(x_0 . y_0)$ a direction. Therefore, the direction of the tangent line of the solution curve passing through the point $(x_0 . y_0)$. The Euler method follows this direction until the next interpolation node. The Runge-Kutta methods consider more points between $(x_0 . y_0)$ and the possible next point $(x_0 + h. y_1)$ of the curve, and depending on the appropriate choice of these additional points we get more accurate value for y_1 so then more exact approximations. Among the different orders which depends also on this mentioned points for this study the 4th order method has picked. The common formulation is as follows;

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$
$$k_1 = f(t_i . y_i)$$
$$k_2 = f\left(t_i + \frac{1}{2} h . y_i + \frac{1}{2} k_1 h\right)$$
$$k_3 = f\left(t_i + \frac{1}{2} h.\ y_i + \frac{1}{2} k_2 h\right)$$
$$k_4 = f(t_i + h . y_i + k_3 h) \tag{13.3}$$

However, there are also existed 5th and 6th or even higher order of the method. For this study the calculations are parallel as the ODEs are connected with each other [30,118]. These calculations would have simulated the counterfactual world.

For the factual world we need a SLM-63, i.e. that ODEs system of equation must be transformed to a SDEs system of equations. For this purpose and the aim of this study we must consider the external forcing and the intensity of the it [108,120,126]. Therefore, the previous chaotic dynamical systems must accept some noises. These random noises supported by the random attractor of stochastic Lorenz model [SLM] which can be represented as the following three SDEs:

$$dx = \sigma(y - x)dt + \delta_1 vx\, dw \quad .$$

$$dy = \rho(x - y - xz)dt + \delta_2 vy\, dw \quad .$$

$$dz = (xy - \beta z)dt + \delta_3 vz\, dw \tag{14.3}$$

Then the previous deterministic model is perturbed by linearly multiplicative noise in the Ito sense, with $W_{t.}$ a Wiener process i.e. the stochastic forcing which is a stationary white noise process as mentioned before and v > 0 the noise intensity (0.5 in this study). The $(\delta_1 . \delta_2 . \delta_3)$ is identified by equal values of √(1/2). The other parameter values are the same as original LM-63 [124,125,126]. Consequently, this leads us to our factual world with its corresponding random noises.

There are several analytical and numerical methods to solve Stochastic Differential Equations. However According to [107] the traditional and most important are the Ito and Stratonovich calculi. These also called as Ito SDEs and Stratonovich SDEs. It is important to note that the "SDEs describe systems in a path-wise fashion although the parameters of SDEs and their corresponding FPE are linked". *This let to estimate SDEs by using FPE* [107,126].

All of this sorts of studies seeking for gaining a numerically stable model by adding some damping as in this case of study [107,126,120]

According to [107] using data assimilation methods in the evolving realm of stochastic climate model is a frontier of this sort of studies as " this increase the ensemble spread" which this by itself gives better adjustment between observations and model forecast. Although the *Kalman filters* are among the most used.

For the aim of this study and also for gaining more exact results nonlinear Ensemble Klalman Filter is used here Among other KFs. Although, it has strong background in the literature of the study as it is based on Bayesian framework and conditional probability and also on the

hidden Markov process. As it is mentioned before KFs are belongs to the data assimilation techniques. Thus it's better to give a brief overview of the issue as kernel of the study.

3.8- Data assimilation

Data assimilation is a statistical method for estimating the true state of a system (in this study is Lorenz-63 dynamic system evolving in time). It works by "merging various measurements irregularly distributed in space and time, with a prior knowledge of the state". It has been widely used in several contexts such as image reconstruction, weather prediction or wildfire modelling.

According to [127,129] "data assimilation is the common name given to the techniques that combine numerical models and measurements in order to obtain an improved estimation of the state of a system". The assumed errors- which is potential in the lots of branch of science and in weather and climate science as well- for both models and measurements is defined by corresponding statistical distribution. The aim is to combine the information of models, measurements and also their uncertainties to achieve a more enhanced estimation. The application of this technique ascends to many practice like, weather forecasting, oceanography, and air quality etc. therefore we going *to estimate the state of the system by combining the observations or measurements along with model simulations and their related errors as well* [128].

As integration either numerical or analytical yields the solution to the given model it easily prone to uncertainty as we are doing an approximation and not an exact solution or *realization* in terms of modeling. This makes the use pdf in evolution of the time (FPE) reasonable. In fact, with the aid of FPE for the "model state" we can extract information about the most likely estimate of the model state as well as its uncertainty. "Since **the PDF contains all statistical information** it can be considered the optimal solution to the estimation problem". [128,129].

Data Assimilation and its inverse methods commonly represent this pdf via *statistical moments* or an *ensemble of model states.* After that they normally look for the *mean* and *maximum likelihood* as their *estimators* with the associated *covariance* representing uncertainty. Here we consider the time evolution of the model state through LM and SLM as chaotic, dissipative and diffusive dynamical models and show how this problem can be solved using Ensemble Kalman Filter (EnKF).

3.8.1- Bayesian Filtering and framework

The main difference between EnKF and KF is that EnKF is implemented to the nonlinear dynamical systems against linearity of the KF. Although the EKF is also used for this purpose of nonlinear systems but for lower computational ends. The fact is that all of these methods are rooted in the *"Bayesian recursive estimation"* to state estimation [129,127,128]. Since commonly we are encountering Markov process in this contest therefore we can define its corresponding probability as

$p(X_k|X_{k-1}.y_{1:k-1})$.

This is equivalent to $p(x_k \mid x_{k-1})$.

For the equation of the dynamical system and the likelihood as $p(y_k \mid x_k)$.

In addition, it is presumed that initial condition is existed and known which is identified as the *prior pdf* of the state $p(x_0) \equiv p(x_0 \mid y_0)$. In the end Bayesian estimator create the posterior pdf as p ($x_k|y_{1:k}$) $p(x_k \mid y_{1:k})$ of the state vector.

For doing so we need two steps *forecast* and *analysis*. The former with

$$p(x_k \mid y_{1:k-1}) = \int p(x_k \mid x_{k-1})\, p(x_{k-1} \mid y_{1:k-1})\, dx_{k-1} \qquad (15.3)$$

This actually is the *Chapman-Kolmogorov* equation and yield the posterior PDF by using prior one. And the latter with

$$p(x_k|\, y_{1:k}) = \frac{p(y_k \mid x_k) P(x_k \mid y_{1:k-1})}{\int p(y_k \mid x_k)\, p(x_k \mid y_{1:k-1})\, dx_{k-1}} \qquad (16.3)$$

This is for updating the prior pdf using the likelihood $p(y_k \mid x_k)$.

This can be achieved from the equation of measurement state using the Bayes rule as shown above.

As the Bayesian estimator is a conceptual solution in most practical cases there is not exist an analytical solution. In fact, its practice is only for linear equations of measurement and observation. Also when x_0, $\{w_k\}$ and $\{v_k\}$ are additive, independent and Gaussian. This is known as the Kalman filter and all the above basics notations and explanations are common in all of the other Kalman Filters.

In more non-technical way, Bayesian estimator is a "*framework for all subsequent recursive filter algorithms*" such as KFs.

"The goal of Bayesian state estimation is to *recursively construct the probability density function (pdf) of the state given the previous state and a new set of measurements*". Put on the Bayesian theorem and by means of pdf The filter uses two steps: the *forecast step* and the *analysis step*. This is also common in KFs. For the system model, which has known noise

process, the model will predict the pdf of the new state from the pdf of the previous state for the *forecast step*. Here this applied on the SLM-63. To enhance the state's pdf estimation, the *analysis step* integrates the current measurement, with known measurement noise [130,128,129].

3.8.2- The ensemble Kalman filter (EnKF)

This method was published for the first time in Evensen (1994). This is an almost complex *sequential data assimilation method*. It is used especially for data in higher dimensions. In fact, it is a *Monte Carlo approximation of the Kalman filter (KF).* The significance is that here, *true covariance matrix in the KF is replaced by the sample covariance matrix computed from the ensemble*. This is in accordance with our dynamical model with its Hamiltonian and canonical distribution of its vectors. Therefore, it could be implemented very efficiently in this study.

In a general view about the nonlinear kalman filters we can say; he unscented Kalman filter delivers, much better results than the extended Kalman filter for nonlinear systems and also by a non- expensive computation. However, it needs sigma points of 2_{n+1}. This leads to an expensive computation in dynamical systems with large temporal and special scales.

But the EnKF solved these problem by tolerating an ensemble members or an empirical amount of samples. As in the case of canonical ensemble in classical statistical mechanics here the size of an ensemble is chosen much smaller than the amount of states which is needed in large-scale dynamical systems. Moreover, as EnKF is an approximation using Monte Carlo method the selection of these *ensembles are random* and not deterministic. In fact, *these ensemble members are samples from the pdf of the true state*. This last property completely answers to our need to a stochastic computation [128,129,130,135].

Error covariance matrix i.e. $p_{k|k-1} \in R^{n\times n}$ which is common in algorithm of KFs in the case of EnkF is not explicitly calculated. For gaining this aim, the matrices $p_y \in R^{m\times m}$and $p_{xy} \in R^{n\times m}$ are calculated.

The Model of this study especially in the case of SDEs is nonlinear and discrete. Then in the case computing EnKF for our final result all of them can be briefly represented as follows;

$$x_{k+1} = f(x_k.u_k.k) + w_k \qquad (17.3)$$

$$y_k = h(x_k.u_k.k) + v_k \qquad (18.3)$$

Where nonlinear functions of. f and h are dimensionally mapped as

$f: R^n \times R^p \times R \longrightarrow R^n$

$h: R^n \times R^p \times R \rightarrow R^m$

and represents system equation and measurement equation respectively.

w_k and v_k are representing the noise process with the characters as follows;

$w_k \approx (0. Q_k)$

$v_k \approx (0. R_k)$

Where Q_k and R_k are known covariance matrices of the noise process.

The expectation values of the noises are as below;

$E[w_k w_j^T] = Q_k \delta_{k-j}$

$E[v_k v_j^T] = R_k \delta_{k-j}$

$E[v_k w_j] = 0$

Here δ_k is the Kronecker delta function as

$$\delta_k \begin{cases} \delta_k = 1 \; if \; k = 0 \\ \delta_k = 0 \; if \; k = 1 \end{cases}$$

Consequently, this guaranteed that the noise processes are white noise then with zero mean also uncorrelated [135.120.129].

However, the algorithm of EnKF can be represented as below which makes the final clarification. This can be before the end step for all method proposed in this study [135,127,136]. The algorithm below and most of the above formulation is derived from [136]. EnKF has a three steps algorithm, beginning, forecast and analysis. These are written down as below(however it is worthy to note that the notations represented here is derived from [136] but there is no restriction therefore there exist different notations but generally all is the same):

1. $\left[\hat{x}_{0|0}^{(1)} . \hat{x}_{0|0}^{(2)} \dots . \hat{x}_{0|0}^{(N)}\right]$, this makes us a matrix of N initial ensemble members which is the initial state of the filter.

2.forcast steps with

$$\hat{x}_{k|k-1}^{(1)} = f\left(\hat{x}_{k-1|k-1}^{(i)} . u_{k-1} . k-1\right) + w_{k-1}^{(i)} \tag{19.3}$$

this transform the ensemble members and

$$\hat{x}_{k|k-1} = \frac{1}{N} \sum_{i=1}^{N} \hat{x}_{k|k-1}^{(i)} \tag{20.3}$$

This is the ensemble mean. This mean estimates the forecast state.

3. this step is the critical an almost long phase of the filtering so let make it more clarify.

a. the predicted measurements is computed by

$$\hat{x}_k^{(i)} = h(\hat{x}_{k|k-1}^{(i)} . \; u_{k-1} . k) \tag{21.3}$$

b. the mean of the predicted measurement is compute is computed by

$$\hat{y}_k = \frac{1}{N} \sum_{i=1}^{N} \hat{y}_k^{(i)} \tag{22.3}$$

The covariance of predicted measurement is calculated by

$$p_{yk} = \frac{1}{N-1}\sum_{i=1}^{N}\left(\hat{y}_k^{(i)} - \hat{y}_k\right)(\hat{y}_k^{(i)} - \hat{y}_k)^T + R_{e_k} \qquad (23.3)$$

$$R_{ek} = \frac{1}{N-1}\sum_{i=1}^{N}(v_k^{(i)})(v_k^{(i)})^T \qquad (24.3)$$

also required cross covariance is calculated by:

$$p_{xyk} = \frac{1}{N-1}\sum_{i=1}^{N}\left(\hat{x}_{k|k-1}^{(i)} - \hat{x}_{k|k-1}\right)(\hat{y}_k^{(i)} - \hat{y}_k)^T \qquad (25.3)$$

The common kalaman gain in EnKF is calculated by

$$K_k = p_{xyk}p_{yk}^{-1} \qquad (24.3)$$

The estimation of the ensemble mean for the analysis step which is the end of the process is;

$$\hat{x}_{k|k}^{(i)} = \hat{x}_{k|k-1}^{(i)} + K_k(y_k + v_k^{(i)} - \hat{y}_k^{(i)})$$

$$\hat{x}_{k|k} = \frac{1}{N}\sum_{i=1}^{N}\hat{x}_{k|k}^{(i)} \qquad (25.3)$$

Note: I in all of the above is 1…. N.

The evidence for above formulation and the model and also the whole process of the calculations is the same as the Bayesian estimator with a tinge of difference which is discussed in chapter of results.

3.9 **Data**

Since this study has general view to the considered problem, the general types of parameters has selected. These data are included in *mean temperature* (as necessary condition), *net radiation flux* (as sufficient condition) and the snow fall data. The period of study i.e. the time interval has been selected for 30 years of DJF from 1987-2017.

3.9.1- why Net Radiation flux

The net radiation flux is calculated as radiation at the surface. This is because of the fact that there was a need to select from the different levels of the prepared data of NOAA. Then for simplicity and saving time of the study ground level was selected. Ground level since the parameters of the study was temperature (air) which is sensed up to 2 meter from ground while the net radiation flux which is calculated by the four series of data from the equation of Net radiation flux (Rn) [137].

Rn = SW↓ − SW↑ + LW↓ − LW↑

Where $SW\uparrow = \alpha SW\downarrow$ and $LW\uparrow = \epsilon\sigma T_r^4$ are thermal long wave and thermal shortwave radiation flux respectively. Also the $SW\downarrow$ and $LW\uparrow$ are the solar long wave and shortwave respectively. These equations display the connection between the reflected shortwave radiation and albedo and also the longwave upward radiation and the emissivity and then the radiative temperature. This selection for the reason that the net radiation at the surface is the result of a "combination of different radiative processes" and directly effects the temperature (air) which also is one of the main driver of the climate system and atmosphere. Another reason is that they are general parameters and is suitable for the up to down view of the study. As it is illustrated before the expandability of the LM-63 by Fourier series expansion for instance made it possible to add more parameters and explore the case of study more ever than this study.

The raster maps of sequential data of time interval the above mentioned dada along with the snow equivalent water for the snow depth are processed in the GIS environment. Then the corresponding DJF data in the same environment. The next step was the selectin of the region which because of the resolution of raster data (1.75 × 1.75) is laid between the N 35 ; 45 and E 25 ; 65 as below figures (1-3..3).

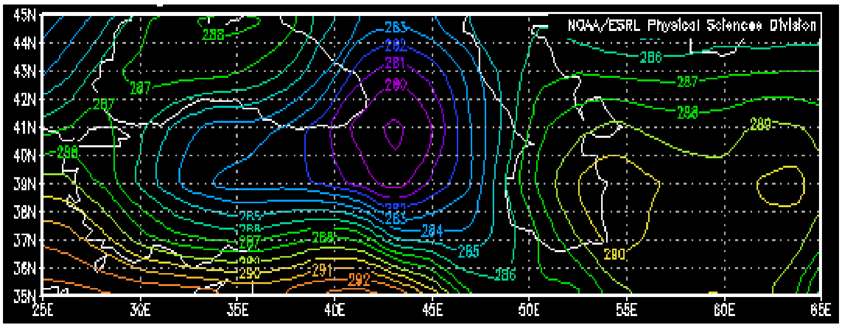


Fig.1.3- 4 daily mean temperature (air) (K) for coordinates of 25E;65E and 35N:45N and level of 2.00. time range: Jan 1 1987 to Jan 1 2017. Max= 295.977 and min= 278.727. (NCPT/DOE AMP Reanalysis II). This map is brought down hear as an evidence of the region of the study and also the total distribution of selected parameter (air) in all of the period. Mazandaran province as the dabbed name of the tittle of the study is located in the sought part of the Caspian Sea.

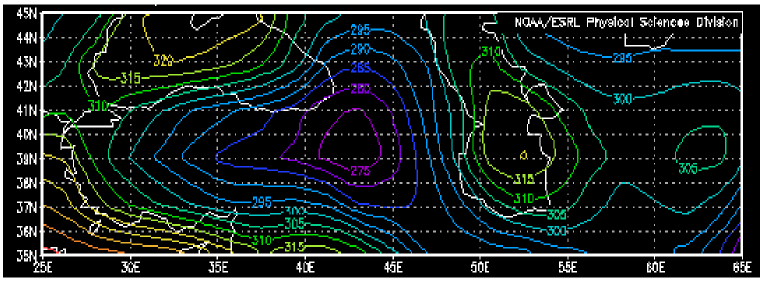


Fig.2.3- 4 daily mean radiation flux(Wm^{-2}) for coordinates of 25E;65E and 35N:45N and level of 1000 Hpa. time range: Jan 1 1987 to Jan 1 2017. Max= 350.095 and min= 271.095. (NCPT/DOE AMP Reanalysis II). This map is brought down hear as an evidence of the region of the study and also the total distribution of selected parameter (air) in all of the period. Mazandaran province as the dabbed name of the tittle of the study is located in the sought part of the Caspian Sea.

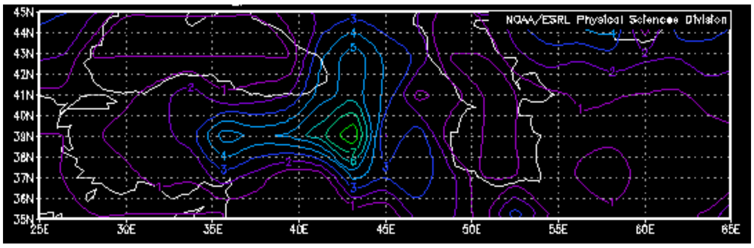


Fig.3.3- 4 daily mean water equivalent of snow depth (Kgm^{-2}) for coordinates of 25E;65E and 35N:45N and level of 1000 Hpa. time range: Jan 1 1987 to Jan 1 2017. Max= 16.4173 and min= 0.0. (NCPT/DOE AMP Reanalysis II). This map is brought down hear as an evidence of the region of the study and also the total distribution of selected parameter (air) in all of the period. Mazandaran province as the dabbed name of the tittle of the study is located in the sought part of the Caspian Sea.

Because of the lack of the Mazandaran weather stations to complete the period of the study and also the fact that the station with more than 30 years are for station of Babolsar, Ghaemshahr, Ramsar and Gharakheil are distributed in the region so inefficiently. Namely two firsts and two second are very near two by two while very far from each other. this made the interpolation or similar approach very impractical. Although even with a deep consideration by the aid of Professor Hooshang Ghaemi it was reveal that for the data of snow fall we have not the related data in early years and this make us our sound data to less than 20 years.

Therefore, we forced to use *satellite data* for this case of study. And because of simplicity to access and also for more time interval the parameters are selected from NCPT, most of them from NOAA satellites. Due to pre-preparation there was no need to do image fustian. As is said in the chapter of theoretical background according to the 2016 report of extreme event attribution of national science academy of the united states there is an undeniable need to better

data fusion techniques to gain more practical results from almost 50 years of satellite data. However, it was one of the main task of this study, it will be a plan for future studies to combine it with our simple or more complicated chaotic model. This is due to the fact that great efforts are made to have a sound Epistemological, physical and mathematical foundations. And this because against the report of national academy of US and according to Palmer, Stott and lucarini the main problem is in the model not in our data.

Another important thing to note is that in this study two worlds of factual and counterfactual are considered and compared instead of having a long time of ***unseen data*** which are prepared more based on mathematical functions and relations and not on the real observation and dividing them to two 50 years as instance. The reason of this was the fact that none of this sort of study can be practical because simply the climate is a chaotic dynamical and physical system then is not predictable with deterministic approaches. In the common way this issued that according to the law of large number and in enough long time the system reaches to it equilibrium i.e. uniformly spread of its particles but it is not the case always as Boltzmann propose in its box model of the universe which says equilibrium is not the end fate of the system so there exist recurrences over and over again which also called Poincare recurrences in an long enough time. So this is the universe with random fluctuation.so predict by the fluctuating random past then what is the most likely event that formed out of chaos so the question is what is the most probable event which made out of chaos. However, in the new theory there is no reverse at all and thing happens at one then go on and so forth.

And another fact is that based on the clear and tidy principle of statistical physics determining an equilibrium time interval is on the knowledge of *relaxation time* of the considered system so who one knows the relaxation time of earth climate system. According to Landau in the book of *statistical physics* (1970), Relaxation Time is "the time within which the system reach to its statistical equilibrium"[131].

Although this is a fact that the LM models i.e. 63,84,96 and 2006 are able to allow you have more parameters to interact for a particular case of study. This is allowable for instance by the *Fourier series expansion*.

Therefore, the main reasons for the selection of most general parameters can be listed as follows:

1- These parameters intrinsically contain all the other sub-parameters.
2- These parameters are the source and main controller of other parameters.
3- Almost all the studies in literature in the context of climate change and attribution have a great uncertainty about the source or cause of the particular extreme event and simply

said "this extreme event have a strong relation to the *forcing of all"*. This means finding the exact cause is Almost impossible!

4- The chaotic climate model is in evolution basically and one of the main purpose of thi study was do an effort for the development of this models.

3.10- Phases of the study

Hitherto it is tried to make the subject sound and clear. This aim has reached by proposing a concise background of the case study. This background is consisting of philosophical, epistemological, mathematical and physical basis of the subject. After that tried to bring the case down to the practical issues and explore in the literature of the attribution and detection and also in the related and non-related extreme events.

These exploring and studying causes to reach the point that, our understanding of the problem is on the wrong way and as lucarini et al (2014) declare that "Equilibrium methods allow investigating many properties of GFD flows but at this point we cannot ignore anymore the elephant in the room i.e. the fact that the dynamics of the climate system cannot be assimilated to an inviscid and unforced GFD flow". Based on this fact and more which is discussed in detail in previous chapters it decided to go on in a different way. Then toward a non- equilibrium thermodynamics by the well- known chaotic, dissipative and non-linear dynamic model of the Lorenz-63.

Although at first glance it may be said that, this is a very simple model and its main practice is in the meteorology not climatology! But please notice to the fact that now we are doing models which is far from the natural fact of the climate system. Also we are doing this by non-chaotic, non- dissipative, equilibrium based, deterministic instead of probabilistic models and then we discussed and conclude with them even do our future plans! Please dare to say " what on earth we are doing".

So this is becoming a world-wide claim in the scientific societies to turn the table and do a more efficient approach. However, there are lots of institutes and organizations which are researching on the traditional ways and insist on it, this study because of its fundamental and established background which took more than one year to be complete, insisted on the chaotic and probabilistic nature of the climate system.

From now on the main problems raised as there is no global model for this sort of study so this turned our case study to be an experimental afford in extreme even attribution studies. And another problem was that most of the studies in chaotic behavior of the climate were left in

theoretical basis not practical. Therefore, this makes this study more motivated and unique. And also another problem was that the long weather station data of Mazandaran province were defected and the distribution of the stations with more than 30 years of climatological data were not well-distributed. Consequently, forced to put this data away and just focus on the satellite data. For having sound fused data sets there were two ways. Select from the related sensors and fused them by software such as GIS or ENVI and select from the prepared data set such as NOAA or ECMWF. The later selected because of time limiting and general point of view for the study. Also for simplicity and general view the data set of radiation and temperature and water equivalent of snow depth were downloaded from NOAA NCEP-DOE Reanalysis-2: NCEP/DOE AMIP-II Reanalysis (Reanalysis-2).

The daily mean data for DJF were used. However, it needed to do some process in GIS to prepare the data for the observation part of the model to verify the simulated results. Actually the role of the data here was the comparison and bring the initial condition for the model to simulate.

Therefore, at first the net radiation flux for the all of time interval i.e. 30 years were calculated and mapped by the GIS. Then the resulted maps which discrete i.e. made for every year one by one fused to have sequence of data for DJF. This also calculated and mapped for the temperature and snow water equivalent. These process made three sequences of 2700 days of DJF (winter time) of related data set. Although it is important to note that because of the main purpose of the study which was *chaotic climate model improvement* and used the designed and modified Lorenz-63 model for the attribution of climate change to the extreme snow fall of Mazandaran province. For this purpose, the following steps by MATLAB and MATHEMATICA and also with the aid of Maple have been executed.

For counterfactual world the classical Lorenz-63 model with its ODEs was executed. For factual world the modified Lorenze-63 model with its SDEs was executed. After that the two models along with the observations were assimilated. The assimilation was Also executed by the Ensemble Kalman Filter which is a nonlinear filter in accordance with the main model. Since the Kalman Filters are basically on the conditional probability and Bayesian methods, this allowed to compute the related Likelihood for the extreme snow falls to the net radiation flux and mean temperature as the sufficient and necessary causations respectively according to the causality theory by pearl (2000).

Chapter IV

Concluding remarks

4.1-Warm up

This chapter is going to gather up all the discussed issues, considered points and related rather sophisticated calculations. As it is tried to the whole of the study be on the sound theoretical basis and foundations, this is also continued in this chapter to conclude the considered theoretical basis, illustrated method by implementing them on the case of study.

The theoretical kernel of the method of this study were the concepts of dissipation, chaos and stochastics. The computational kernel of the method were the concepts of ODEs, SDEs, FPE, EnkF and the deterministic Lorenz chaotic dynamical model (1963) along with its stochastic chaotic dynamical model i.e. SLM-63. To have a sound combination of all mentioned concepts to gain a sound results there was a need to bring all of them in a more compact way by highlighting the more critical concepts such as EnKF and conditional probability of the Bayesian approach.

The Lorenz-63 is a chaotic dynamical system which is extensively used in the meteorological field because it clarified the unpredictability of the weather conditions more than 5 days (as Tim Palmer illustrated in his lecture about the point by the Russian doles).

In this study it is used because there is no universal chaotic nonlinear, non-equilibrium, dissipative, random, stochastic and probabilistic climate model. In fact, there is a critical need for this kind of models because the extreme weather and climate events are extensively rising in the last decades and the current deterministic, conservative, equilibrium climate models have nothing to do in practice. As most of them are working on frequency and deterministically compute their differential equations and try to make the non-linear nature of the climate system linear just to solve their mathematical differential equations! As it is illustrated in the chapter 2 this approaches and frameworks have a far distance from the real case of the extreme climate events.

The cause' of Nietzsche's Will to Power learned us to look at the problem from up to down i.e. find the problem, clarify it, seek for general causes then bring them down to details to achieve a logically sound interpretation. The causality of Pearl (2000) learned us to seek for the conditional and necessary and sufficient causation to verify our interpretations instead of p-

value of the conventional methods. The next section as the same of the chapter 3 would be on the trace of physics, mathematics and logic of the method

4.2- the results of LM-63 and SLM-63 for the factual and counterfactual worlds

4.2.1 initial calculations

In the nonlinear system of three ODEs the Lorenz-63 model yielded us the required counterfactual world. The reason is that the forcing we need as a driver is existed explicitly in the model. And also that is because the model is derived and truncated from Naiver–Stock equations as illustrated in chapter 3.

This was accomplished with the initial condition provided by the prepared PDF of parameters of mean temperature, net radiation flux and snow falls (snow equivalent water) for the DJF of 1978 to 2017. However, for an initiative testing it is preferred to do a simulation with the original data sets at first and also for curiosity. The resulted graph had shown interesting fractal structures. Seemed more strange than usual strange attractors of LM-63. This has done as bellow.

The preparation of data was performed by GIS system of software. Then the resulted raster data yield us three sequence of DJF for the required 30 years of observations. Note is that for the fact that our parameters of causation are the net radiation flux and temperature (air) (in daily mean) the ground level was selected. This is also due to time and space saving of the study. This can be case for the other future researches.

Theses selectin of time interval is usual in climate studies but as it is said in the end of the chapter 3 this also seriously subject to question as who one knows the relaxation time of the climate system to say that after 30 years of time interval or even more or less the system reach to its equilibrium point even generally?!however for instance in [13] or identical research tried to measure the return time of the climate system mainly by the CO_2 radiative forcing which yield 25 to up to 125 years which has a great time uncertainty.

The common approach of treatment with a deterministic model of ODEs are implementing numerical method instead of analytical methods because of practicalities that mentioned in chapter three. The first experimental step was the testing the model for all 10000 days of the period on the 3-D Lorenz-63 system. For this purpose, the Runge-kutta numerical method

solved the system of equations in 4^{th} order. The resulted graph of the model is as below (Fig.1.4).

Fig 1.4. the resulted realization of the LM-63 model by implementing the Runge-Kutta method in 4rt order. The values for the initial conditions of the three vectors of (X, Y, Z) are gained by the prepared data sequences by GIS environment. Note that the two graphs are the same just with the different view. Also note that the central part of the graph is the common strange attractors of the LM-63 then the exponential and circular part of the is deviated from the main part. The symmetrical behavior of the model is also identified by the zero value at the center. The x axis is corresponding for the mean temperature (k) (air) up to 2 meter from the ground (from -300 to +300 k), the y axis is belonged to the snow depth (Inch) and the z axis is for the net radiation flux Wm2 (from -300 to +400). However, with the changing in the place of the values or even the amount of them there was no change in the shape of fractal structure of the graph.

Fig.1.4 showed the primarily resulted graph of the 3-D lorenz-63 model for the 3 parameters of temperature (air), net radiation flux and snow equivalent water (snow depth wo). The time interval is equal to 10000 days of all the period of study. The chaotic behavior of the system is clear. Also the exponential behavior after final deviation is made because of fluctuation of lyapunov exponent. That is lyapunov exponent identify the expansion of the stage attractor fractals. Also it is showed that the exponent is positive i.e. $\lambda>0$. The time evolution for this exponent is presented in the figure (2.4).

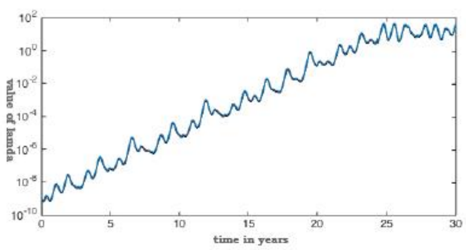


Fig. 2.4 the time evolution of lyapunov exponents corresponding to the dynamical trajectories for 30 years of the whole period of the study which is an evidence of the chaotic and exponential geometry of fractal structure of the simulated model of LM-63 by thee initial condition of the original data.

This guaranteed the chaotic behavior of the system and then of the parameters which defined the required vectors or sets of values. The $\Delta t = 1$ for each day of the whole period then it is the number of time increment in the model.

The resulted graph of figure 1.4 is illustrating a world without forcing and just with the internal variability. please note that this is just the experimental performance on the LM-63 model by the original set of data while the LM-63 and SLM-63 is performed by the corresponding PDF (FPE) values. The latter is on the foundations of state space models and discrete nonlinear models with additive white noise along with the "recursive Bayesian state estimation" as the most fundamental [136]

4.2.2- SLM-63 and LM-63 as parallel factual and counterfactual worlds.

The parallel LM-63 gives us the counterfactual world and solving with EnKF give us an optimal solution for the large dynamical system such as the system of this study. From the figure (3.4) the fact of growing amount of the net radiation flux at the surface is clear. However, as it is showed in the graph, when the net radiation flux has the higher value the snow precipitation and the mean temperature(air) are experiencing their lowest values! This point can also be a topic for the future.

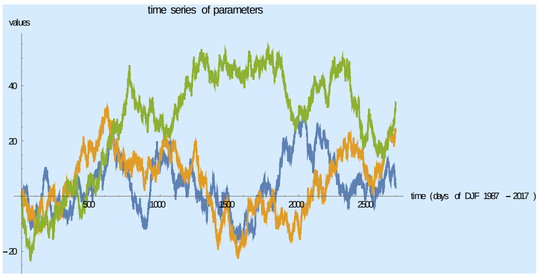


Fig.3.4 – the time series distributions of original observations for three parameters in winter time(DJF). The blue line is the time series distribution the snow precipitation (equivalent water of snow depth). The orange line for the mean temperature(air) and the green is for net radiation flux as the causing force for the extreme snow in the region of study. It is identified from the graph that however the primary days the three line are moving along with each other in the middle of the period the radiation line diverge from the normal line and has very high positive value while the two others are experiencing a moving along path approximately.

There was a need to transform above distribution to a PDF distribution of three parameters. This is calculated by the kernel density smoothing since it treated as an integral transformation. The resulted graph is represented below figure (4.4).

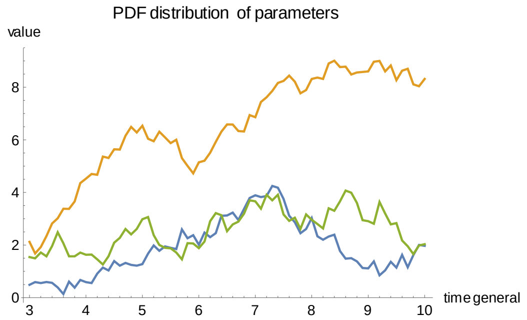


Fig. 4.4 PDF distribution of the three in used parameters i.e. blue line for snow depth equivalent water, the green line for net radiation flux and the orange line for mean temperature (air). the initial conditions for the corresponding FPE of the model is derived from this distribution.

For the counterfactual part the LM-63 with the initial condition of PDF of observations (figure.3.4) is executed. Which showed little dense fractal structure that is because of the initial condition values which here were the values of PDF of observations(Fig.4.4.). However, in the previous condition(Fig.1.4) with the changing in the place of the values or even the amount of them there was no change in the shape of fractal structure of the graph here with a change in value you have another fractal structure.

For the factual world however there was a need to add white noises to the original LM-63 to make it SLM-63. The white noises were with Gaussian distribution with zero mean and known deviation. The graph of the noises is as bellow, figure (Fig.5.4).

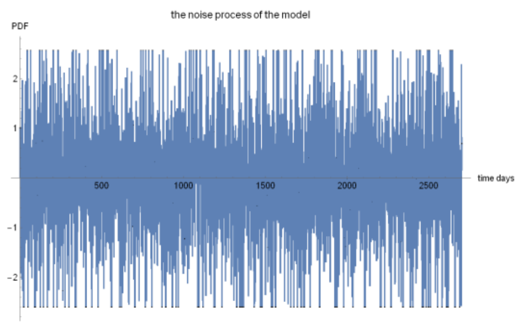


Fig.5.4 - the graph of white noise process that has the role of making the perturbation for the SDEs of the model.as it is clear the distribution of the noise process is a Gaussian distribution which is centered at (-1) to (+1). These noises are the correspond to the generation of random attractors in the SLM model.

The noises in the figure (5.4) are uncorrelated namely their covariance is equal to zero. This help to have a Gaussian distribution for the mutually exclusive noises. However, the nature of intensity of noises may or may not be Gaussian retribution but the results of the filter is fairly promising in this study and also in several identical cases [143]. Although the reason of this success is not clear yet.

To realize the counterfactual and factual corresponding PDF(FPE) the EnKF is implemented on the LM-63 with its unperturbed FPE and also on the perturbed SLM-63 as the counterfactual and factual worlds. In fact these are the realization of ω over Ω and Ω'. Then are showing the probability density of three defined parameters in counterfactual and factual (left to right) worlds respectively. In this respect the graph can be interpreted as he resulted graphs of the model in the forecast/measurement state are as below in Figure (6.4).

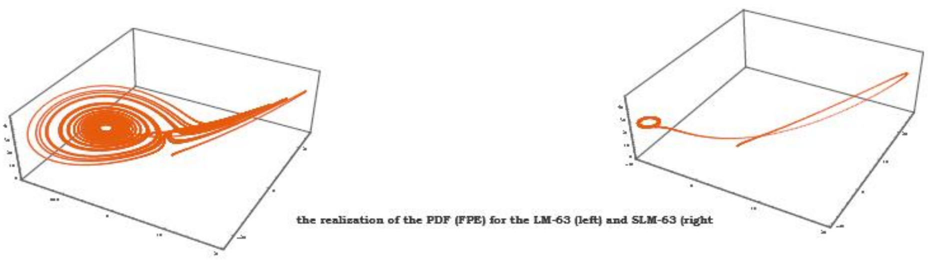


Fig 6.4- two graphs of the resulted LM-63 and SLM-63 left and right respectively for the forecast step. These graphs are produced by the PDF(FPE) values of the X_t and y_t . the symmetric behavior then equilibrium manner of the counterfactual world is showed in the left graph and the non-symmetric behavior then non-equilibrium manner of the factual world is showed in the right. The less dense graph against the figure (1.4) is because of the truncated and transformed data as PDF as initial condition and also for the added noises in the factual world. This also caused for the predictability and less chaotic behavior of the factual model than counterfactual model. The important result of the graph then is the power of SDEs system of equation and also the Bayesian nonlinear filtering as EnKF. In fact, this is the forecast step of the ensemble kalman filter.

Here the task was this; associate a label $\omega \in \Omega(w_t\, v_t)$. This derive the stochastic model in the context of EnKF and the related SLM-63. The role of these noises were to perturbed the observation or measurement in the SDEs system and propagate the system in its evolution interval. However, the time is implicitly implemented to the model by the model itself. The SDEs which produced this result is solved actually by the EnKF's intrinsic conditional probability and it roots in the Bayesian produced (chapter 3) random process. Here the strange attractors of the previous model with the original data turned to the random attractors which is perturbed and produced by the stationary white noises with the covariance given by identity

matrix i.e. Q and R. The $(\delta_1.\delta_2.\delta_3)$ is identified by equal values of √(1/2) as the intensity of forcing and $\sigma(v.w) = 0.5$. The significant point of these two figure is the fact that for the counterfactual world we encounter with the PDF in the equilibrium point and that showed that reaching to equilibrium and have internal variability and predictability with the current deterministic approaches is in the condition of having a system without external random perturbations while it is impossible. In figure of SLM-63 we can see the non-equilibrium nature of the PDF which by itself identified the dissipative, random, and also chaotic nature of the climate system in our general view.

The forced ODEs system of the above deterministic chaotic model turned into SDEs system of equation by adding w_t and v_t of white noises. Here the task was this; associate a label $\omega \in \Omega(w_t\, v_t)$. This derive the stochastic model in the context of EnKF and the related SLM-63. The role of these noises were to perturbed the observation or measurement in the SDEs system.

Also it must be note that the SDEs system defined in this study is stationary .it means that the PDF (which here is the FPE actually) of the state vector i.e. X_t has no dependence on the initial condition after sufficiently long time t namely initializing step at X_0 and also on the t by itself. This PDF (FPE) is the mean over Ω of the sample measure is calculated by each realization ω (solution in this context) of white noises mentioned above $(w_t\, v_t)$. The graph of the noise process for 2700 days is identified in the figure (5.4).

About the SLM-63 system each sample measurement is supported by a random attracter instead of deterministic attractor or strange attractor of LM-63. This attractor has a fine structure (fractals) and was also dependent on time. But this PDF(FPE) of the unperturbed Lorenz-63 as showed in (Fig 1.4) was for counterfactual world has showed the strange attractor or Lorenz attractor smoothly.in the factual world however the nature of the PDF is identical but its fractal is affected by the forcing parameters.

4.2.3- Conclusions of the implementing EnKF for two parallel worlds

This step is the calculation of the PDF (FPE) for the models of LM-63 and SLM-63 corresponding to the counterfactual and factual worlds showing forecast step in the algorithm of EnKF in the context of Lorenz-63**.** The trajectories in the previous graphs are made by initial conditions od PDFs of the three parameters of mean temperature, net radiation flux and snow equivalent water. The kernel density estimation (K (x, t)) is applied to the ensemble of simulation obtained from forcing to estimation of the corresponding PDFs. As mentioned in

chapter 3 the whole process of SDEs analysis needs integration either analytical or numerical then there is need to integral equation and the transformations. The EnKF are doing this task here and yield numerical integration to the SDEs system here. These results are explained in the following by two trajectories in figure (7.4). these trajectories are showing the evolution of the simulation in the ensemble space through the observation time interval. Note that this evolution in time is corresponding to the LM-63 and SLM-63 as the counterfactual and factual worlds respectively.

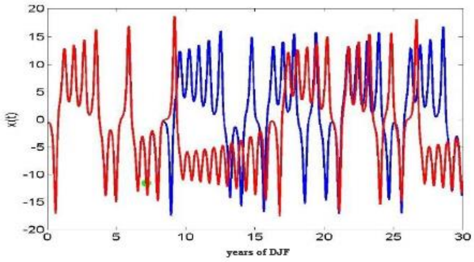
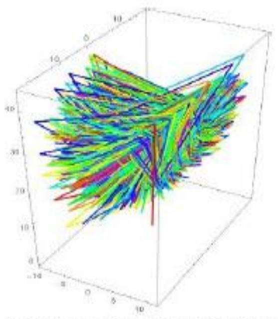


Fig. 7.4 - the trajectories for the forecast step of LM-63 (red) and SLM-63(blue) in comparison with each other. The time is in years to have more smooth trajectory. The trajectories are meeting each other in just two trivial areas. This is identifying the difference between the LM-63 and SLM-63 in their time evolution.

Until here the forecast step or measurement step of the EnKF is implemented on the situation of the case study. To reach to the aim of this study there is a need to gain the causal probability. This would be fulfilled by the analysis step of the EnKF. As mentioned before this phase do this task as a maximum likelihood in a few sub-steps. First is combination of PDFs of the true states with the observation all in one. This is performed for all 2700 days of DJF (winter time) of the time interval of the study in its LM-63 and SLM-63 i.e. counterfactual and factual parallel worlds. The figure (8.4) displayed the results of the combination in two very dense figure of the three PDFs with the observation.



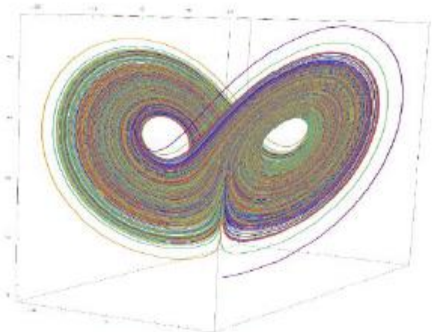
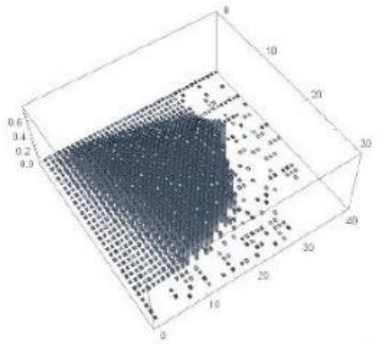


Fig. 7.4. the displayed figures of the analysis step of the EnKF for the LM-63 and SLM-63 as the counterfactual and factual worlds left to right respectively. As it is clear from the left figure the resulted graph for the LM-63 in the context of EnKF filtering showed a very dense but non-smoothing shape and irregular with an ambiguous iteration in its evolution in time. Although in the factual world, EnKF showed a smoothing result with a clear iteration of the dynamical system. this comparison also showed the fact that the white noises which added to the model in its SLM-63 part is the causing of this smoothing and symmetric analysis. The coarse appearance of the left figure is the evidence of unpredictability of the occurrence of the extreme snow fall in Mazandaran province in a deterministic world but predictability of the event in the world of stochastics. The smooth and symmetric appearance are the prominent evidence.

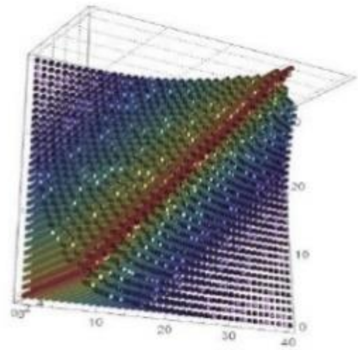
Please here note that the logic behind this results and is interpretations for the aim of this study is on the basis " the fractal geometry of nature" by B.B Mandelbrot (1982). So the may be called it ambitious aim of the future is on the Fractals representation of the dynamical system. which it seems that is a fairly certain road to solve the problem as it basically is on the chance and probability the readily displaying the noised in the phenomena that makes it unpredictable or so! The difference between these two PDF (FPE) generated graphs is that they are identifying the existence of an area of the phase space that is more likely to be reached in the factual than in the counterfactual world (Fig. 4.4). The note is that this result is due to the EnKF formulation which is illustrated in chapter 3 that is actually was a combination of the Monte-Carlo and Bayes' theorem. Then is initially and fundamentally random and probabilistic such as the climate by itself.

According to algorithm of the EnKF the covariance matrix has an important role in the process of filtering. Since it is the identification of the amount and shape of the variations of the corresponding parameters in the phase space. Another reason is an evidence of the efficiency of the process on this case study of extreme snowfall of Mazandaran province. Figure (8.4) is

the proof of this expressions for the counterfactual and factual worlds as below.



the realization of covariance matrix of LM-63 with the observation points



the realization of covariance matrix of SLM-63 with the observation points

Fig.8.4 the 3-D plot of the covariance matrix corresponding to the LM-63 (left) and SLM-63 (right) in the context of EnKF for the counterfactual and factual worlds respectively. The significance point of these figures is the distribution of the extreme observations in two worlds. This is shown that the SLM-63 has very more points than the LM-63.

Figure (8.4) is a clear evidence of the Advantages of the stochastic process on the deterministic proceed for the extreme attribution study in the context of climate change. The symmetric distribution of the produced points in the analysis step of the Ensemble kalman filter along with the observation points is an evidence of the efficiency of the SLM-63 model for the factual world than the LM-63 for the counterfactual world.

Then we are searching for sequences from both worlds in which an event did accrue, by identifying the leading direction of projections of the sequences in the phase space. Consequently, the goal is to identification of the maximum likelihood of the trajectory of the sequences in the phase space to be reached in the factual world than in the counterfactual one (if there existed). Therefore, it is clear that the probability of the event in the former is found to be higher than in the latter, namely $p_1 > p_0$. then the occurrence of an event in this context can be define mathematically as $\{max_{\{0 \le t \le T\}} \hat{\delta}' y_t \ge u\}$,where δ' is the unit vector of forcing. The resulted graph of this process is showed by the figure (8.5). The main interpretation of this graph is the existence of the random noise for the constant forcing in the SLM-63 that caused the more likelihood of extreme snow fall occurrence in this part of the model than in the part of LM-63. The first as the factual and the second as the counterfactual worlds.

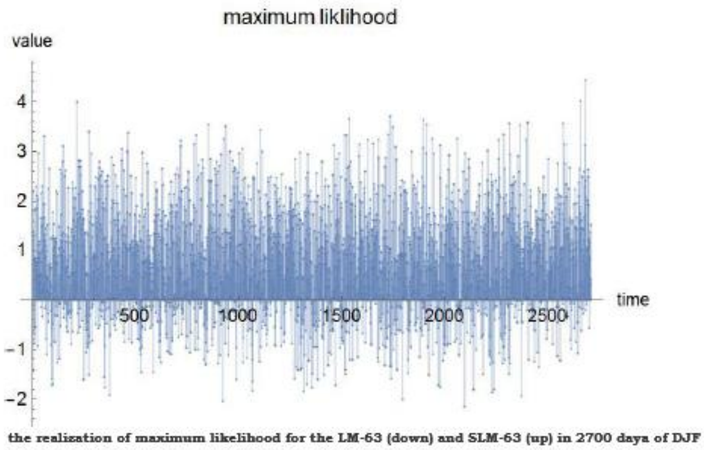


Fig.9.4. the time series of the values of maximum likelihood of the occurrence of extreme snow fall in Mazandaran province in factual (up) and counterfactual (down). The main interpretation of this graph is the existence of the random noise for the constant forcing in the SLM-63 that caused the more likelihood of extreme snow fall occurrence in this part of the model than in the part of LM-63. The first as the factual and the second as the counterfactual worlds. The amount of predictability and attributability to the forcing which is represented as the net radiation flux is also clear. This is actually a display of the log likelihood result of the proposed likelihood function in chapter three? More illustration in the text.

This is the enlightening form of the process because it from a causal perspective. And that is due to the fact that the related necessary and sufficient probabilities for the causation of the events are positive. Figure (9.4) is resulted from the trajectories associated with the event in two worlds appear to have distinct features. The positive part as the factual and the negative part as the counterfactual worlds. The amount of predictability and attributability to the forcing which is represented as the net radiation flux is also clear.

The likelihood (maximum) function is transformed of the probability for a single point in theoretical way. Here this is defined as ML_0 and ML_1 for the counterfactual and factual world which is the basis for the calculation of three required probabilities of necessity, sufficiency and necessity and sufficiency in the context of causal attribution theory. This is done by the help of Bayes theorem since the conditional relation between the three parameters of the study is the main foundation and also the end is the causal attribution to the forcing as the net radiation flux by the connection of mean temperature to the extreme snow fall.

For clarification the end formulation is brought here as;

$$L = \frac{p(y|x)p(x)}{p(x|y)} \tag{1.4}$$

The numerator is the model promise and denominator is the end result. This rooted in the theorem of total probability and is a inferential approach that concentrate on x. in fact taking the logarithm of the likelihood function in its Gaussian assumption yield this result such that the maximum likelihood of the Fig.9.4 is actually the log likelihood results of the proposed likelihood function. for more refer to the chapter 3.

As illustrated in the previous sections the main theoretical concepts of this study are LM-63 and SLM-63 dynamical models which was selected to bring us the factual and counterfactual worlds. The resulted and depicted proposed figures of were going to do a compression in respect of these two near real time worlds. The critical part was the corresponding trajectories of the state vector x_t simulated under factual conditions namely in the presence of the additional forcing. This is also perturbed by the white noises to stochastically guarantee the required situation. Then this trajectory along with the observations y_t that is showed by the figure (4.4) in T=2700 (0f DJF days) are combined by the process of the EnKF to brought us the log likelihood of the occurrence of extreme snowfall with the defined forcing in the context of chaotic model of Lorenze-63.

The EnKF is used to assimilate these observations into a model of factual world p=1 consequently it matches the forecast/measurement model used in the simulation step then a reconstructed trajectory is obtained from corresponding analysis step. Also the Maximum likelihood which defined as $ML_{o.1}(y_t)$ to attribute to the state vector x_t. this is showed in the figure (8.4).

Note that the required assimilation is done in the counterfactual and factual models to obtain corresponding trajectories, from the same observation namely one time with the SLM-63 and observation and one time with the LM-63 and observations. The corresponding likelihoods are LM_0 and LM_1 for the counterfactual and factual respectively. the figure (8.4) is illustrating this point.

The local differences between the estimated likelihood of two trajectories of LM-63 and SLM-63 are showed in the figure (8.4). consequently, they are the instant inconsistencies between the associated factors of the ML (x, y). the cumulative difference is also identified by the figure which is a distinct gap between the results of the factual and counterfactual models.

The last step of the whole calculation for this study was the computation of the probability of occurrence with respect to data assimilation results (log likelihood) in the context of EnKF. This is done by the change in the pearl (2000) definition of necessity and sufficient and necessity and sufficient conditions for the causation then attribution.

Figures of (10.4) is the probability graph of causation according to the PN, PS and PNS by their corresponding mean with a remarkable grows over time for the probability of necessity.

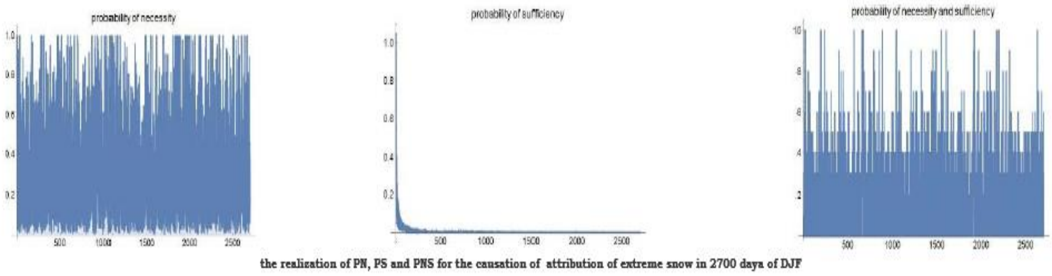


Fig.10.4 the computed probability of necessity, sufficiency and probability of necessity and sufficiency over the time interval of 2700 days of DJF from 1987-2017 for the extreme snow fall of Mazandaran province with respect to their corresponding forcing. These three graphs are showing the time distribution of the related probabilities over the time period. The probability of necessity possesses the greater values the probability of sufficiency has the most least values and the probability of necessity and sufficiency has the lesser values. More on text.

The probability of sufficiency showed the most least values due to the fact that this probability according to the pearl (2000) is the probability of causation without the causes that lead to the occurrence of the event in its probability of necessity. Therefore, in this case of study this is the probability of extreme snow fall in Mazandaran province without the defined forcing (net radiation flux as the forcing of all; as it is common in attribution studies).

The probability of necessity is then the probability of extreme snow fall in Mazandaran province in the presence of the defined forcing. This probability with its greater values is as an evidence of the whole calculations in this study along with the two others as its complement actually.

The probability of the necessity and sufficiency is actually the probability of extreme snow fall in Mazandaran province with and without the defined forcing for this study. However, it still has very greater values than probability of sufficiency due to the great influence of the values of the probability of necessity. The mean values of theses probabilities are also computed in the table below.

PN	PS	PNS
0.8978	0.1942	0.4519

Table.1.4 the mean values of the computed PN, PS, and PNS. These probability values are doing so as the p-value in the conventional approaches of Attribution of climate change.

The meaning of the table (1.4) is that the extreme snow falls of Mazandaran province with the probability of necessity value of 0.8978 is attributable to the fluctuation of the intensity of the defined forcing (net radiation flux) in relation with the mean temperature(air) for the period of this study. It also is attributable to the other forcing in its value of probability of sufficiency in the value of 0.1942 and to its value of probability of necessity and sufficiency in the value of 0.4567.

These all together is the end result of the computation for this case of study about the attribution of extreme snow fall in Mazandaran province. So then according to this study the extreme snow falls of Mazandaran province in the 30 years of 1987 -2017 of DJF (winter time) is attributable to the fluctuation of the intensity of net radiation flux that caused the fluctuation of the intensity of mean temperature (air) in the general view of this study with its experimental but modified dynamical model of Lorenz-63 in the context of the EnKF.

4.3 The ending remarks

This study had two main purposes. The first one was the practice of the illustrated method instead of complex climate models in real case with a general view of parameters, physical, mathematical approaches. However, the cause and casual theory of the Pearl (2000) is the main thread which gather all of them. And the second one was the a more ambitious aim and that is also for now and future.

From the illustrated points and theoretical basis of this study it can be interpreted that the randomness and noisy behavior of the climate system is clear and it is clear that because of the complexity of the system not the dynamic and no the physics of the system can be illustrated by the conventional deterministic methods as the climate system is an open and extended system not a closed system and easily prone to noises.so then the approach which can be helpful in practice is interring to the world of fractal geometry since it is rooted on probability not on the determinism and more importantly provide an iterated but random shapes. This by itself explain the fact that the *causes are there but the intensity is differing randomly.* Then one the main tools which is very practical in this context is the development of the numerical solutions of stochastic differential equations instead of ODEs or PDEs.

CHAPTER V

Discussion and suggestions

5.1- Gather up all

Climate change and its field of study is growing very dramatically fast. The core of this progression is on the numerical solutions of differential equations as the core mathematical form of defining the climate system as a dynamical system. This form of mathematics brings the main part of study in the realm of physics not geography! Then if geographer wish to study this part of the geographical area which is closely related to the human life on the planet then must use the physics of the problem therefore must learn it or release it completely to the physicists! The reason is that the mere working on the data of the climate and do not attend to the contribution of physical process that produced this data is a serious mistake in tackling with the problem.

Therefore, there is an undeniable need to have a comprehensive model to have exact solution or near approximation for the problem at hand. Here the mathematics said this is not possible because the system is very complex and this complexity made it improvable. So the only way is concentrating on the produced data. Physicists here said this is not the case as the basic physical rules which is describing the system of climate must define the system more correctly to the transforming the system to mathematical form be also more correct and then selection of the solution to the problem also would be more correct! This is the fundamental basis and motivation of this study.

Consequently, it is tried in the previous chapters to bring the problem down to its foundation the solve it in a general view. That was because every sound research in natural sciences require a sound epistemological basis which is related to the problem at hand, a sound physical basis which is rooted in its epistemology and also the problem itself the also a sound mathematical basis which is related to the former and later respectively.

As far as this study penetrated on the literature of the case study i.e. climate change in general and attribution of it in particular, the above points in the more of the studies are seriously absent. It seemed that every one in every paper just focus on a particular dynamical or mathematical issue separately and also with a non-clear epistemology! Most of the studies is trying to improve the numerical solutions for the related differential equations. It is true that the dynamical system of the climate is defined by the differential equations as it is a dynamical

system and that is it is evolved in time, that makes it dynamic i.e. with motion. But wat is the physics that caused the motions?!

Since the current models of the climate systems is seriously lacking in this respect this study went toward the initiative an intuitive point at first step and seemed for a sound basis, physically! This seek went to the world of mechanics, such as Lagrangian and Hamiltonian, to the world of statistical physics and mechanics, closed system canonical ensembles, equilibrium and entropy and so on and so forth. Here it is found that the return time or period, the internal variability, the study of extremes is on the basis of this classical not modern physical basis and that is the root of the problem.

The reason of call it a problem is that the climate system is not closed but open then interactive and outer-active, with increasing entropy, then non-equilibrium. The noses in the system which are intrinsic to the system made a serious problem in defining the initial conditions which is the first point in solving differential equation corresponding to the model of the system. when the noised IS on the system and is a part of the system why treat it separately!? The system is not conservative the all the rules of the classical mechanics are fail down in relation to the problem and is not helpful anymore! [140].

5.2- SDEs or ODEs?

SDEs are the extension of ODEs in the realm of noises. In fact, the errors do not calculate separately but is a part of the system. According to the Brownian motion this noises are random sense the motion is random as well in a microstate scale. Then According to the butterfly effect theory and cause' and also causality the randomness is a natural and inseparable part of any natural system.

Climate system is a natural system it must be random and governed by probability. Then we are faced with a stochastic not deterministic system. Then we need to move on the house of SDEs and random process instead of ODEs.

According to the Mandelbrot (1982) most of the nature is irregular and random so the geometry of it is fractals not the Euclid. The simplest and more practiced model for the start of a novice researcher which has this property and the property of the last paragraphs is the Lorenz-63 chaotic dynamical model.

In fact, in this study the motivation of the usage of this model for a long time of period is that the SDE system of it is also practiced in literature and plenty of paper and books worked on it even the Mandelbrot in his book of " fractal geometry of nature".

According to the above discussions walking on the path of fractals is the way of success for climate modeling which also is a low cost approach.so the main task is developing numerical methods for solving SDEs in a large scale system such as climate. So if you can display the suitable fractal of the case study in this context you are home!

5.3- A brief discussion on the method

The main method presented here were ODEs system vs. SDE system as the factual and counterfactual worlds. The reason for this parallel selection of worlds instead of having a long time period such as 100 years is that the real presented data of climate system is not so long as this and the Brownian motion in the system do not allow to an exact or near production of UNSEEN data! Another reason is that weather forecasting in current models is on the 5 days of certainty then how the climate models in current generations can predict the far future! Then period of the study is separated to two parallel words of 30 years (DJF of 1987-2017) in near real time with certain near real data.

Data assimilation technique is proposed to assimilate the PDF(FPE) of the model and PDF of the observations i.e. daily mean temperature, daily mean net radiation flux and daily mean snow equivalent water. The selection of this types of the data was on the basis of the general view of the study and then the most general datasets were selected. As mentioned in the section of (1.5) this is because of the top-to-down perspective of the study and also the Nietzsche's perspectivism which is illustrated in chapter 2.

EnKF was implemented as data assimilation technique to provide the required maximum likelihood based on the mentioned PDFs. *This study was trying to find causal links between the change in net radiation flux as climate forcing (forcing of all) and the change in mean temperature (air) with the extreme snow fall of Mazandaran province.* This is called detection and attribution in climate studies as well. Therefore, it searched for the probability change of the occurrence of extreme snow fall in Mazandaran province with change in mentioned forcing i.e. net radiation flux that caused the change in mean temperature (air) and that caused change in probability of extreme snow fall in the region.

Such as statistical mechanics the ensembles concept is used. Then the probability of occurrence of extreme snow in an ensemble of simulation by LM-63 and SLM-63 models. These models were representing the observed climate conditions which simulates the probability of occurrence of extreme snow fall in the real world i.e. SLM-63 with the probability of occurrence of the same event in a parallel world of ensemble of its simulations i.e. LM-63 as an alternative. In literature these worlds dabbed as factual and counterfactual worlds.

If p_1 and p_0 be the be the probability of occurrences in the factual and counterfactual world respectively. Then the probability of necessity, the probability of sufficiency and probability of necessity and sufficient for the causation of extreme snow fall of Mazandaran province can be derived as pearl (2000). Although in this study this is turned to the likelihood instead of probability in the context of EnKF as the data assimilation technique.

5-4 **The future development**

This part is dedicated to the future enhancement of the whole process in general and the method of this study in particular. However, this is actually not possible to overcome all the area, a brief review would be helpful.

Climate change research and its detection and attribution for the extreme weather and climate events is in progress. However most of the current research is on the mere statistical method developments this study argued that with this approaches such as fingerprint methods or storyline methods which are regression based and frequency based methods a great part of the problem left unseen! That is there is need to better understanding and modeling of the physics of the process. Although it is clear that understanding the whole process of the climate system is very time consuming and may be impossible but working on the classical theories and expect a modern result seems strange! Event with the aid of super computers.

Therefore, as the modern physics declare the non-equilibrium and entropy increase of the whole universe, the climate system as a tiny part of this whole is sharing this property. Then we are dealing with a non-equilibrium, dissipative, chaotic, random and probabilistic system which uncertainty is a part of this system not something in outer space. Then as in it local and global view this study shown that the stochastic approaches and developing the numerical methods along with a serious attention to the "fractal geometry of nature" and climate as a part of it we can consider the problem, in a sound way. However, the significant case study of Lyapunov exponents is an interesting field. Although it is more placed in the deterministic

world of computation, its combination with probability case would have more interesting result for the development of stochastic climate theory and models

5.5 Ending Remarks

The added noises to perturbed the LM-63 were from a source of uncorrelated Gaussian noise. To solve this randomly perturbed system which is also nonlinear the KF is not the case since it is suitable for linear cases of this type models i.e. state space models. Although in this case of study the property of being large-scale is another limitation. EKF can be an alternative as it considers the nonlinearity. But it also has two limitations, first is that it makes a nonlinear case a linear case and not suitable for large- scale system however its use of Jacobian Matrices makes very expensive to computation. UKF with its deterministic selection of sigma points is also not suitable for large scale systems as it uses a fixed error covariance matrix [141].

Although the filter used in this study (EnKF) is on the UKF system but it is designed to large scale systems to estimate their states in the phase space. It picks up a large number of samples called ensembles. This causes to a higher quality of state estimation. Such as its statistical physics (classical) cases these ensemble sizes are very smaller than state numbers of the large-scale system by itself. Another case to add is that these ensemble members which are the PDFs from the forecast state are randomly selected by the Monte-Carlo method. The error estimation is performed in the analysis state by covariance matrix (i.e. error covariance matrix). However, the defects of the EnKF can be illustrated as follows.

The main defects are on the system of perturbations such as; it merely solves underestimation problem for a finite amount of ensembles and this system of perturbation implement additional error in its sampling. Although in current research it is showed that with the use of multiplication noise instead of additional noise, these problems can be improved. The main reason is that these noises are completely a part of the system as its natural cases. Sensitive to ensemble size. Under sampling can lead to filter divergence. Ideas to mitigate this include localization and inflation. Assumes Gaussian statistics, for highly non-linear models; this may not be a valid assumption. However other ensemble-based filter and also particle bases filters tried to solve the problem of EnKF filter but not have had remarkable enhancements. This can also be an interesting case for future studies [138,141,142].

END THE ROSE THE RED ONE

Abstract

The climate change is an active field of study in the geographical, physical, mathematical or even other field of science such as sociology or even psychology. The aim of this study was to do attribution of climate change to the extreme snow fall of Mazandaran province in the northern part of Iran and southern shores of the Caspian Sea in the vicinity of Alborz mountains. The area is not prone to extreme snow fall or even snow fall then this phenomenon is had great damages to the infrastructures of the region. The study is performed on the time interval of 1987-2017in winter time (DJF). Corresponding calculation is done for two parallel worlds of counterfactual i.e. without external forcing and factual with external forcing in the context of stochastic climate modeling. This is done by the chaotic dynamical 3-D model Lorenz-63. Then the two worlds of study are defined on the basis of LM-63 and SLM-63 as deterministic and stochastic climate models. Fakher- Planck equation had the role of implementing time evolution of the PDF into the model. The conditional probability and Bayesian framework is the preliminaries of the method of this study. The model is belonging to the space state models and Bayesian recursive estimation. These all is the basis of the EnKF as nonlinear filtering approach to the nonlinear dynamical model of this study. It is tried to bring down all the related situation associated with the issue on the basis of sound mathematical, epistemological and physical foundations. All the computations are done on the environment of GIS, Matlab, Mathematica and Maple. Then Conterfactual causal theory of pearl (2000) is used as the evidence of verification for the whole process. The final results showed that the extreme snow of Mazandaran province is attributable to the climate forcing defined for the study 0.8978 in its PN causation, 0.1942 in its PS causation and 0.4519 in its PNS causation.

Key words: Attribution, stochastic process, Lorenz-63, Ensemble Kalman Filter (EnKF), Counterfactual causal theory.

چکیده

تغیر اقلیم و پیامد های آن یکی از ضمینه های فعال در علوم میباشد که در شاخه های مختلف علم مانند جغرافی، فیزیک، ریاضیات و دیگر شاخه ها مانند جامعه شناسی و حتی روان شناسی مورد بررسی قرار گرفته است. استان مازندران در شمال ایران و سواحل جنوبی دریای کاسپین و در جوار رشته کوه های البرز قرارگرفته است. این ناحیه از کشور ناحیه ای برف خیز نیست و ریزش برف سنگین در این منطقه از موارد نادر در اقلیم آن به شمار می آید. از همین رو خسارت های زیادی را به زیر سا خت ها وارد میسازد. این مطالعه در راستای بررسی نسبت دهی تغییر اقلیم بر بارش های برف سنگین مازندران انجام شده است. دوره زمانی آن هنگامه زمستان یعنی ما های دسامبر، ژانویه و فوریه برای سال های ۱۹۸۷–۲۰۱۷ انتخاب شد. محاسبات مربوطه برای دو دنیای موازی واقعی با اجبار و غیر واقعی بدون اجبار در ضمینه مدل سازی تصادفی اقلیم انجام شده است. این امر با کمک مدل آشوبی دینامیک Lorenz-63 انجام شده است. سپس دو دنیای موازی واقعی و غیر واقعی به ترتیب توسط SLM-63 و SLM-63 به عنوان مدل تصادفی و مدل معین ایجاد شد. معادله Fokker- Planck نقش اعمال سیر زمانی تابع چگالی احتمال را به درون مدل ایفا کرد. احتمال شرطی و چهارچوب بیزین مبانی ای مطالع به شمار می آیند. مدل متعلق به دسته مدل های شرایط فضایی و تخمین بیزی برگشتی می باشد. همه ی این موارد بنیان های EnKF به عنوان یک فیلتر غیر خطی برای مدل غیر خطی این مطاله می باشند. تلاش شده تا همه موارد مربوطه بر اساس مبای ریاضی ، فیزیکی و روش شناسی مد نظر قرار گیرد. محاسبات این مطالعه توسط نرم افزر های GIS, Matlab, Mathematica و Mapleانجام شده است . سپس تئوری علیت پرل (۲۰۰۰) به عنوان مدرک تایید همه پروسه مورد استفاده قرار گرفت. نتایج انتهایی نشان داد که برف سنگین استان مازندران میتواند به اجبار تعریف شده در این مطالعه نسبت داد شود ۸۹۷۸/. در سبب احتمال ضروری، ۱۹٤۲/. سبب احتمال لازم و ٤۵۱۹/. سبب احتمال لازم و ضروری.

واژگان کلیدی: نسبت دهی، پروسه های تصادفی، Lorenz-63 ، فیلتر کالمن انسنبل(EnKF)، تئوری علیت

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